

SAMPLE — a model of format and depth, not an answer

A Fuel-Limited Dogleg Recovers Most of the Survival on a Two-SAM Transit Leg

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Abstract—A notional aircraft must fly a 160 km transit leg past two medium-range surface-to-air missile (SAM) sites on a fuel load that allows at most 185 km. We model each site’s lethal zone as the intersection of its detection range and its missile engagement zone, convert its kill chain into a hazard per kilometer of exposure, and score one-waypoint dogleg routes by their probability of survival P_S . The straight leg is the shortest route and nearly the most dangerous one ($P_S = 0.167$). Offsetting the waypoint north clears the second site’s zone from an offset of 27.1 km and raises P_S steadily, but the fuel limit caps the offset at 46.4 km. The recommended route, a 46 km dogleg of 184.6 km, survives with $P_S = 0.754$; its residual risk comes entirely from the first site, whose zone the fuel limit does not let the route clear. The choice holds for per-shot kill probabilities from 0.20 to 0.35.

Index Terms—Aircraft survivability, hazard model, route planning, surface-to-air missile.

I. INTRODUCTION

A PENETRATING aircraft rarely has the fuel to fly around every threat. The planner’s question is how much of the available fuel to spend buying distance from the threats that matter most. This paper answers that question for a deliberately small notional case: one transit leg, two SAM sites, and one fuel limit.

The contribution is a scored comparison of dogleg routes under a survival model built from the course lessons, ending in a recommended route and its residual risk. Section II reviews the models, Section III states the scenario and threat model, Section IV gives the route-planning method, Section V reports the results, Section VI discusses limitations, and Section VII recommends a route.

II. BACKGROUND AND RELATED WORK

Survivability is an aircraft’s capability to avoid or withstand a hostile environment; this paper treats only its susceptibility half, the chance of being hit [1]. The kill chain gives a site’s per-shot kill probability as the product of its conditional links, and because independent shots multiply survival, the logarithm of survival adds along a route [2]. A radar’s detection range grows only as the fourth root of the target’s radar cross section (RCS) [3], and the RCS itself changes with the aspect the

Sample paper for the ECE 495 Final Project, Fall 2026, written by the instructor as a model of format and depth. Its scenario is a notional toy—one transit leg past two SAM sites of the Lesson 34 demo’s public medium-range class—and is unrelated to the Final Project scenario; none of its sites, routes or numbers carries over.

aircraft presents [4]. A site can engage only where it both detects the aircraft and can reach it, so its lethal zone is the intersection of its detection contour and its missile engagement zone (MEZ) [5]. With an additive risk per unit length, route planning becomes a shortest-path problem that trades path length against exposure under a fuel limit [6]. Jamming can shrink a radar’s detection range [7], and GPS denial degrades the navigation that a planned route assumes [8]; Section VI returns to both.

III. SCENARIO AND THREAT MODEL

Positions are (North, East) in km. The aircraft flies from $A = (0, 0)$ to $B = (0, 160)$ (Fig. 1). Two notional medium-range SAM sites sit south of the straight leg: S_1 at $(-10, 50)$ and S_2 at $(-20, 110)$. Both use the public medium-range class of the course’s route-planning demo [6]: a nose-on detection range of 35 km, a MEZ radius of 45 km, a per-shot kill probability $p = 0.25$, and one shot per $\Delta s = 20$ km of track inside the lethal zone. Detection, not missile reach, binds, so each lethal zone is a circle of radius $R_L = \min(35, 45) = 35$ km [5]. The fuel load allows a leg of at most $L_{\max} = 185$ km.

Surviving ℓ km of lethal-zone track means surviving $\ell/\Delta s$ independent shots, so the survival is $(1-p)^{\ell/\Delta s} = e^{-\lambda\ell}$, with the hazard

$$\lambda = \frac{-\ln(1-p)}{\Delta s} = 0.0144 \text{ km}^{-1} \quad (1)$$

for both sites [2]. A route that crosses ℓ_i km of site i ’s zone survives with

$$P_S = \exp\left(-\sum_i \lambda_i \ell_i\right). \quad (2)$$

Holding each detection range at its nose-on value is an aspect-blind simplification that Section VI examines.

IV. ROUTE-PLANNING METHOD

Both sites lie south of the straight leg, so the candidate routes bend north: a one-waypoint dogleg $A \rightarrow W \rightarrow B$ with $W = (h, 80)$, the leg’s midpoint displaced h km north. Its length is

$$L(h) = 2\sqrt{80^2 + h^2} \text{ km}, \quad (3)$$

so the fuel limit $L(h) \leq 185$ km allows $h \leq 46.4$ km.

For each h , the exposure ℓ_i is the length of the route inside site i ’s lethal circle, summed over the two segments.

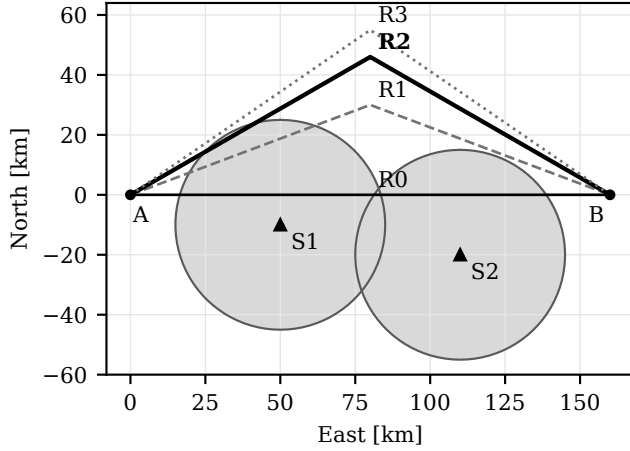


Fig. 1. The leg from A to B , the two lethal zones (radius 35 km), and the four candidate routes. R2 (bold), the fuel-limited choice, clears S_2 's zone and crosses only the northern edge of S_1 's; R3 clears both but exceeds the 185 km limit.

A segment from $\mathbf{p}_0$ to $\mathbf{p}_1$ is inside a circle of center $\mathbf{c}$ and radius R_L for the parameters $t \in [0, 1]$ that satisfy

$$\|\mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0) - \mathbf{c}\|^2 \leq R_L^2, \quad (4)$$

a quadratic in t whose two roots, clipped to $[0, 1]$, bound the segment's chord; the chord's length times $\|\mathbf{p}_1 - \mathbf{p}_0\|$ is the exposure. Equation (2) then gives P_S . A short Python script swept h from 0 to 70 km in 0.1 km steps, recorded L and P_S for each offset, and selected the offset with the highest P_S within the fuel limit, rounded down to a whole kilometer for the waypoint. Four routes are reported: the straight leg R0 ($h = 0$), an intermediate dogleg R1 ($h = 30$ km), the fuel-limited choice R2 ($h = 46$ km, the largest whole-kilometer offset within the limit), and R3 ($h = 55$ km), which clears both zones. The sweep evaluated 701 offsets and Table I keeps only the four that carry the argument; a sweep over two choices at once reads best the same way, as one row per value of the first choice with the best value of the second and its P_S . A grid search with Dijkstra's algorithm [6] would consider routes this one-parameter family cannot; Section VI discusses what that could change.

V. RESULTS

Table I and Fig. 1 give the four routes. The straight leg R0 spends 124.5 km inside the two lethal zones and survives with $P_S = 0.167$. Bending north removes S_2 first: its zone is cleared from an offset of 27.1 km, so R1 meets only S_1 and raises P_S to 0.525 for 10.9 km of extra track. Because both sites share one hazard, P_S depends on h only through the total lethal length, which never grows with h across the feasible offsets; the fuel-limited choice is therefore the largest feasible offset, flown at a whole-kilometer waypoint as R2, with $P_S = 0.754$ at 184.6 km. R3 clears both zones ($P_S = 1.000$ within the model) but needs 194.2 km, 9.2 km beyond the limit; on this route family, clearing S_1 takes at least 188.7 km. Against R3, R2 gives up 0.246 of survival to the fuel limit, all of it to S_1 .

TABLE I
CANDIDATE ROUTES ($p = 0.25$, $\lambda = 0.0144 \text{ km}^{-1}$)

Route	h [km]	L [km]	ℓ_1 [km]	ℓ_2 [km]	P_S
R0	0	160.0	67.1	57.4	0.167
R1	30	170.9	44.7	0.0	0.525
R2	46	184.6	19.7	0.0	0.754
R3	55	194.2	0.0	0.0	1.000

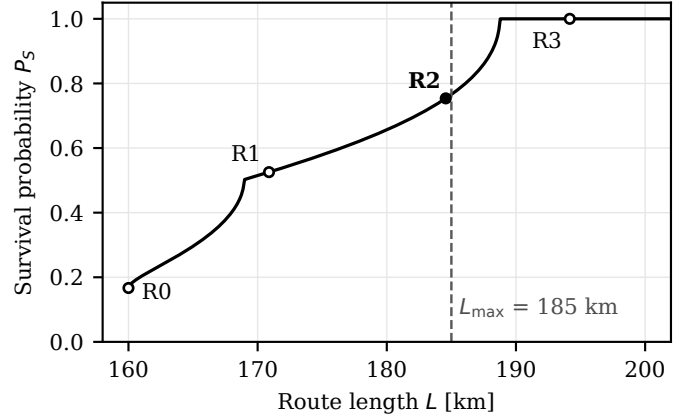


Fig. 2. Survival versus route length over the dogleg family, h from 0 to 70 km. Each step is one lethal zone being cleared; the fuel limit (dashed) falls inside the second, which is why R2 still carries S_1 's risk.

Fig. 2 shows the whole trade. Survival rises with route length in two steps, one for each zone the route leaves: the first ends at 169.0 km, where S_2 's zone is cleared, and the second at 188.7 km, where S_1 's is. The fuel limit falls inside the second step, so the best feasible route pays for its last few kilometers of fuel at the steepest part of the curve.

Near R2 the survival is steep in the offset: P_S is 0.707 at $h = 44$ km and 0.818 at $h = 48$ km, a route of 186.6 km that is just over the limit. The choice of R2 does not depend on the per-shot kill probability: for $p = 0.20$ and $p = 0.35$, R2 remains the best feasible whole-kilometer route ($P_S = 0.803$ and 0.655), while the straight leg falls from 0.249 to 0.068.

VI. DISCUSSION AND LIMITATIONS

These results are a teaching model's promise, not a measured survival. Three simplifications matter most here. First, the lethal zones are circles drawn at the nose-on detection range, but RCS varies strongly with aspect [4]: on R2's two legs each site sees a different aspect, so its real contour is not a circle, and a larger RCS at the aspect R2 presents would enlarge S_1 's zone exactly where R2 passes it [5]. Second, the shots are independent with a fixed p ; correlated or cued shots would change λ , and with it every P_S in Table I. Third, only one-waypoint doglegs were searched; a two-waypoint route that hugs S_1 's circle, or a grid search [6], could lower the exposure within the same 185 km.

The model also leaves out everything but the two sites' shots: the adversary's ability to track the aircraft outside the lethal zones, the return leg, and any threat not on the laydown. Each would add risk to every route in Table I, so the

recommendation below is a floor on the risk, not a prediction of it.

Two operational trends bear directly on R2. A stand-off jammer that shrank S_1 's detection range below R2's closest approach would clear its zone without more fuel [7]. Conversely, R2 sits on the steep part of its survival curve, so navigation error matters: under GPS denial, an inertial drift that flattened the dogleg to a 44 km offset would lower P_S from 0.754 to 0.707 [8].

VII. CONCLUSION AND RECOMMENDATION

Fly R2: a dogleg with its waypoint 46 km north of the leg's midpoint, 184.6 km long, which avoids S_2 entirely and survives S_1 with $P_S = 0.754$ in this model. Do not fly the straight leg ($P_S = 0.167$). The residual risk, 0.246, comes from 19.7 km of track inside S_1 's lethal zone; removing it takes 3.7 km more fuel than the 185 km allowed, or support that shrinks S_1 's zone.

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APPENDIX A
SOURCE VETTING

TABLE II
SOURCE VETTING (EXTERNAL SOURCES ONLY)

Source	What it supports	Vetting (one line)
[1]	The definitions of survivability and susceptibility in Section II	AIAA Education Series textbook, 2nd ed. (2003), by R. E. Ball; commercially published and publicly available; its definitions agree with Lesson 31 [2].