

Homework 4 – Survivability and Mission Planning

Name: _____

Instructions and Grading Policy

Instructions:

HW4 takes you, *individually*, through every Block 4 model once, one question per lesson: the survival metric and the threat map (Part 1), information, routes and tracking (Part 2), and the support package and the navigation threat (Part 3). These are the calculations your Final Project runs at scale; if you can do them by hand, you can sanity-check everything your code produces. Every number here is notional and differs from the Final Project's. You may reference course notes, the readings, and the Final Project handout, but all work must be your own. **Use of GenAI is not authorized for any part of this assignment.**

Grading Policy:

- Each question is worth **1 point**, graded on the boxed entries only. Total possible points: **10**.
- A question with several boxes splits its point equally among them; a resubmission may correct any or all of them.
- Report numeric answers to **three significant figures**, with units where asked. Where a box asks for a label or a route, write exactly what the question asks for.
- If a question does not meet standard, you may submit **one resubmission per question** to recover the full point. A **blank answer does not qualify** — resubmission recovers an attempted answer, not a skipped one.
- **Late policy:** coordinate with the instructor at least 24 hours before the deadline. Non-coordinated late submissions receive a zero, and that zero cannot be recovered by resubmission.

Conventions

Use the Block 4 models and notation throughout.

- Positions are North–East coordinates (N, E) in km; bearings are measured clockwise from North, $\theta = \text{atan2}(\Delta E, \Delta N)$.
- The kill chain is $P_K = P_D P_{T|D} P_{I|T} P_{H|I} P_{K|H}$, with susceptibility $P_H = P_D P_{T|D} P_{I|T} P_{H|I}$ and vulnerability $P_{K|H}$. Independent encounters multiply: $P_S = \prod_i (1 - P_{K,i})$.
- Work every angle noise σ_θ in **radians** where a formula calls for it.

Part 1 — The survival metric and the threat map

Q1. The kill chain (L31). One SAM engagement against the B-21 has the links $P_D = 0.75$, $P_{T|D} = 0.85$, $P_{I|T} = 0.95$, $P_{H|I} = 0.35$ and $P_{K|H} = 0.55$.

- Compute the susceptibility P_H .
- Compute the engagement's P_K .
- Classify each lever as susceptibility reduction (**S**) or vulnerability reduction (**V**); write five letters, in order.
 - A towed decoy that draws the missile's seeker away from the aircraft.
 - Nitrogen inerting of the fuel tanks, so a fragment strike cannot ignite the vapor.
 - An exhaust shield that hides the hot nozzle from IR sensors on the ground below.
 - Duplicate flight-control wiring routed down opposite sides of the fuselage.
 - Armor plate around the flight-control computers.

Sanity check: (a) is a product of four links below 1, so it is smaller than the smallest of them — expect 0.15 to 0.3. (b) is smaller again, by the factor $P_{K|H}$. An (a) near 0.12 put $P_{K|H}$ into P_H ; it is the one link that acts after the hit. For (c), ask of each lever whether it changes whether they hit you or what a hit does to you; being built into the airframe does not decide it.

(a) P_H :

(b) P_K :

(c) **S or V, (i)–(v)**:

Q2. Survival across encounters, and additive risk (L31). A route meets three independent threats. A long-range SAM gets one shot with $P_K = 0.22$. A medium-range SAM holds the B-21 inside its lethal zone for $\ell = 36$ km of track and fires once every $\Delta s = 12$ km, each shot killing with $p_{\text{shot}} = 0.14$. A point defense at the target gets one shot with $P_K = 0.09$.

- The medium-range SAM's hazard, $\lambda = -\ln(1 - p_{\text{shot}})/\Delta s$, per km.
- The route's survival probability P_S (the medium SAM's survival is $e^{-\lambda\ell}$).
- The route's additive risk, $r = -\ln P_S$.

Sanity check: (a) is close to $p_{\text{shot}}/\Delta s$, about 0.01 per km; a value near 0.15 left out Δs . (b) is below every single encounter's survival — expect 0.4 to 0.5; a value near 0.33 is $1 - \sum P_K$, the small- p shortcut. (c) must exceed the sum of the three encounters' kill probabilities (about 0.67), because $-\ln(1 - p) > p$ for every p ; a value below that sum skipped the log.

(a) λ : per km

(b) P_S :

(c) r :

Q3. Horizon versus R^4 (L32). An X-band tracking radar on a 20 m mast detects $\sigma_{\text{ref}} = 1 \text{ m}^2$ at $R_{\text{ref}} = 72 \text{ km}$. At its current aspect the B-21 presents $\sigma = 0.3 \text{ m}^2$. Detection needs both a strong enough echo, $R = R_{\text{ref}}(\sigma/\sigma_{\text{ref}})^{1/4}$, and a line of sight, $d = 4.12(\sqrt{h_r} + \sqrt{h_t}) \text{ km}$ with heights in meters.

- (a) The B-21 flies at $h_t = 45 \text{ m}$. Compute its detection range (the bound that binds).
- (b) Compute the crossing altitude h^* : the aircraft altitude above which the R^4 law, not the horizon, sets the detection range.

Sanity check: (a) is the smaller of your two bounds — expect 40 to 55 km. About 28 km means the radar's mast term was dropped (L11's one-ended horizon); near 100 km means the two bounds were added, though they are two conditions that must both hold. (b) is tens of meters — expect 50 to 100 m — and it must agree with (a): if h^* is below 45 m, the binding bound in (a) was the R^4 law, and if it is above, the horizon.

(a) Detection range: km

(b) h^* : m

Q4. MEZ versus LEZ (L32). A notional SAM's missile averages $v_m = 1.4 \text{ km/s}$ and can fly for at most $t_{\text{max}} = 36 \text{ s}$. The B-21 flies at $v_t = 0.24 \text{ km/s}$. Use the constant-average-speed model.

- (a) The MEZ radius, $R_{\text{MEZ}} = (v_m + v_t) t_{\text{max}}$.
- (b) The LEZ radius, $R_{\text{LEZ}} = (v_m - v_t) t_{\text{max}}$.
- (c) The site launches while the B-21 is 48 km away and closing, and the B-21 turns directly away at launch. How long must the missile fly to catch it? (Compare with t_{max} : does it escape?)

Sanity check: (a) and (b) sit one above and one below the missile's own fly-out, $v_m t_{\text{max}} = 50.4 \text{ km}$. Equal values left the target's speed out; an LEZ larger than the MEZ has them swapped — a closing target meets the missile partway. (c) is tens of seconds, 30 to 50 s; a value near 29 s used the closing speed $v_m + v_t$, but a fleeing target is chased at $v_m - v_t$. A catch time longer than t_{max} means the missile runs out of flight first.

(a) R_{MEZ} : km

(b) R_{LEZ} : km

(c) Time to catch: s

Part 2 — Information, routes and tracking

Q5. Two-sensor CRLB and PDOP (L33). Two of the adversary's bearing-only IRST sites sit at $(N, E) = (0, -48)$ km and $(0, +48)$ km, each with $\sigma_\theta = 0.7^\circ$ (1σ). The B-21 is at $(90, 0)$ km. Each site's bearing row is $\mathbf{h}_k = [-\Delta E_k, \Delta N_k]/R_k^2$ (with $\Delta N_k, \Delta E_k$ from site k to the B-21) and $\mathbf{J} = \sum_k \mathbf{h}_k^\top \mathbf{h}_k / \sigma_\theta^2$; the CRLB ellipse comes from the eigenvalues of $\mathbf{J}^{-1}$.

- (a) The long 1σ semi-axis a of the CRLB ellipse on the B-21, and the direction it lies along (N-S or E-W).
- (b) The pair's PDOP, $\sqrt{2}/\sin \beta$, where β is the cut angle between the two lines of sight.
- (c) The DRMS, $\text{PDOP} \cdot \bar{R} \cdot \sigma_\theta$.

Sanity check: (a) is a kilometer or two — expect 1 to 3 km. The long axis lies along the direction the two bearings measure least: a bearing measures only across its own line of sight. About 100 km means σ_θ went in in degrees; a value with units of km^{-1} took $\sqrt{\lambda}$ of $\mathbf{J}$ instead of $1/\sqrt{\lambda}$. (b) can never be below $\sqrt{2}$ for a pair — expect 1.5 to 2. (c) is 1.5 to 3 km and must equal $\sqrt{a^2 + b^2}$ from your ellipse, to rounding; near 120 km left σ_θ in degrees.

(a) **a and direction:** km, and N-S or E-W

(b) **PDOP:**

(c) **DRMS:** km

Q6. Dijkstra on a $-\ln(1-p)$ graph (L34). A planner's graph has the start S , the target T and waypoints $A-D$. Each leg can be flown in either direction and carries the kill probability p of flying it:

$S-A$ 0.18, $S-B$ 0.02, $A-C$ 0.15, $A-D$ 0.35, $B-D$ 0.45, $C-T$ 0.16, $D-T$ 0.01.

- (a) Weight each leg with $r = -\ln(1-p)$ and run Dijkstra from S to T . Which route does it return? (Write the nodes, e.g. $S-X-Y-T$.)
- (b) That route's survival probability P_S .
- (c) A second planner weights each leg with the raw p instead. What is the true P_S of the route it returns?

Sanity check: (b) is $e^{-(\text{your minimum total cost})}$ — expect 0.5 to 0.7. (c) cannot exceed (b): minimizing $\sum -\ln(1-p)$ maximizes P_S exactly, so the log-weighted route is the most survivable route on the graph. If your (c) is larger, one of the two runs went wrong.

(a) **Route:**

(b) **P_S :**

(c) **Raw- p route's P_S :**

Q7. The weight and fuel trade (L34). A weight sweep on one ingress returns only four distinct routes, each with its length L and additive risk r :

Route	R1	R2	R3	R4
L [km]	380	420	475	540
r	1.20	0.45	0.18	0.04

The planner minimizes the cost $c = L + w r$, with the weight w in km per unit of risk.

- Which route does the planner return at $w = 100$ km?
- Above what weight w does the planner switch from R2 to R3?
- The fuel limit allows at most 510 km on this ingress. What is $P_S = e^{-r}$ of the route you fly?

Sanity check: (a) At $w = 0$ the planner returns the shortest route and at very large w the safest; $w = 100$ km lies between. (b) The switch comes where the two costs are equal — the extra length divided by the risk it saves — expect 150 to 250 km. (c) The fuel limit only rules routes out: of the routes that fit, fly the safest. Expect 0.8 to 0.9; a route that is closest to the limit but over it does not get home.

(a) **Route:**

R1, R2, R3 or R4

(b) w :

km

(c) P_S :

Q8. One EKF update, and observability (L35). The B-21's EKF tracks a stationary pop-up emitter from the own-ship position (0, 0). Its prior is $\mathbf{x} = [N \ E]^\top = [20 \ 48]^\top$ km with $P = 25 I \text{ km}^2$ (5 km in each axis). One RWR bearing arrives, $z = 66.5^\circ$, with $\sigma_\theta = 0.5^\circ$. Run the L35 update, with $\Delta N, \Delta E$ from the own-ship to the prior and angles in radians:

$$H = \frac{[-\Delta E \ \Delta N]}{R^2}, \quad S = HPH^\top + \sigma_\theta^2, \quad K = \frac{PH^\top}{S},$$

$$\mathbf{x} \leftarrow \mathbf{x} + K \text{ wrap}(z - h(\mathbf{x})), \quad P \leftarrow (I - KH)P, \quad h(\mathbf{x}) = \text{atan2}(\Delta E, \Delta N).$$

- (a) The updated estimate (N, E) , in km.
- (b) $\sqrt{\text{tr } P}$ after the update (before it: 7.07 km).
- (c) Is the emitter's range observable (**O**) or unobservable (**U**) from the own-ship's own bearings? Write four letters, in order.
 - (i) A stationary emitter off to one side of a straight own-ship leg.
 - (ii) An emitter moving at constant velocity; the own-ship flies straight at constant velocity, taking a bearing every second.
 - (iii) An emitter moving at constant velocity; the own-ship flies a dogleg.
 - (iv) A stationary emitter dead ahead on the own-ship's straight track, before any turn.

Sanity check: (a) moves the prior by less than a kilometer, and across the line of sight, not along it. (b) One bearing can shrink only the axis across the line of sight; the 5 km along it is untouched, so $\sqrt{\text{tr } P}$ lands between 5 and 5.5 km. A value below 5 km shrank an axis a bearing cannot measure. For (c), ask whether the bearings could equally well come from the emitter at some other range: a better or faster RWR thins the ellipse, it does not shorten it.

(a) (N, E) : km

(b) $\sqrt{\text{tr } P}$: km

(c) **O or U, (i)–(iv)**:

Part 3 — The support package and the navigation threat

Q9. Stand-off burn-through (L36). A stand-off jammer (SOJ) screens a strike package from an X-band acquisition radar with $P_t G_t = 83$ dBW; the package presents $\sigma = 0.05$ m². The SOJ radiates $P_j G_j = 36$ dBW toward the radar over $B_j = 4B_r$ (four times the radar's bandwidth) and orbits $R_j = 90$ km from it. The radar's sidelobes are -28 dB. Burn-through is at $J/S = 10$ (10 dB):

$$R_{\text{bt}} = \left[\frac{10 P_t G_t \sigma R_j^2 B_j}{4\pi P_j G_j (G_r'/G_r) B_r} \right]^{1/4}.$$

- (a) The sidelobe burn-through range, for a package off the jammer's azimuth.
- (b) The mainlobe burn-through range, for a package on the jammer's azimuth (in the strobe).
- (c) A newly located SAM forces the SOJ back to $R_j = 160$ km. The new sidelobe burn-through range.

Sanity check: Work in watts and meters. (a) is ten to twenty kilometers. (b) The mainlobe lets 28 dB more jamming into the receiver, so (b) is shorter than (a) — expect 2 to 4 km; about 0.6 km used $10^{\text{dB}/20}$ where a fourth root of J/S wants $10^{\text{dB}/40}$. (c) The SOJ is farther away, so less jamming reaches the radar: (c) must be longer than (a), but by much less than the $1.8\times$ increase in standoff. A value shorter than (a) went the wrong way.

(a) Sidelobe R_{bt} : km

(b) Mainlobe R_{bt} : km

(c) Sidelobe R_{bt} at 160 km: km

Q10. GPS denial radius (L37). A notional GPS jammer radiates an ERP of 25 W toward the air route. The L1 C/A signal (1575.42 MHz) arrives at $S = -130$ dBm, and the receiver tolerates $(J/S)_{\text{tol}} = 30$ dB. The jammer's one-way free-space loss is $L_{\text{fs}} = 20 \log_{10} R_{\text{km}} + 20 \log_{10} f_{\text{GHz}} + 92.45$ dB.

- The denial radius R_{deny} .
- The route passes $d = 150$ km from the jammer at $V = 900$ km/h. How many minutes does it fly without GPS? (The route crosses the chord $\ell = 2\sqrt{R_{\text{deny}}^2 - d^2}$.)
- A lower-grade INS with a notional drift rate of 20 km/h navigates that chord. What is its position error at the exit?

Sanity check: (a) is a couple of hundred kilometers — expect 150 to 300 km. About 15 km means the path was charged twice ($40 \log_{10} R$, a radar's round trip; the jammer's signal travels one way); about 7.6 km put the ERP in dBW against a signal in dBm. (b) is tens of minutes — expect 20 to 30 min. (c) is a few kilometers — expect 5 to 10 km, the drift rate times the time in hours; hundreds of kilometers multiplied by minutes.

- (a) R_{deny} : km
- (b) Time without GPS: min
- (c) Exit error: km