ECE 495 – Electromagnetic Warfare and Survivability

Block 1: Radar Fundamentals
Lesson 8: Detection Theory

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Where We Left Off

L3 gave us $R_{\max}$. We plugged in some $S_{\min}$ from the data sheet.

But where does $S_{\min}$ come from? It is not a constant — it is a *contract* between detection probability and false-alarm rate.

Today: noise, SNR, P_d, P_{fa}, ROC curves, and integration — the math that decides whether a return becomes a track.

Lesson Objectives

By the end of this lesson, you will be able to:

- Compute the **noise floor** for a given bandwidth and noise figure
- Explain the **detection threshold** and the P_d/P_{fa} tradeoff
- Read a **ROC** curve and identify the SNR that matches a mission requirement
- Quantify the SNR gain from **coherent** and **non-coherent** integration

The Detection Problem

Every radar return is a signal sample buried in noise. Compare the sample to a threshold:

- Above threshold $\Rightarrow$ declare *target*
- Below threshold $\Rightarrow$ declare *noise*

Two error modes:

- **Miss** — target present, threshold not crossed
- **False alarm** — noise alone crosses the threshold

Every detection is a thresholded coin flip.

The Noise Floor: $N = kTB$

Thermal noise sets the floor below which we cannot see:

$$\boxed{N_{\text{dBm}} = -174 + 10\log_{10}(B_{\text{Hz}}) + \text{NF}_{\text{dB}}}$$

- -174 dBm/Hz is kT_0, the thermal floor at the 290 K reference temperature
- Bandwidth B adds $10\log_{10} B$
- Noise figure NF is the receiver's own contribution

Quick example: $B = 10$ MHz, NF $= 3$ dB $\Rightarrow$ $N \approx -101$ dBm.

SNR and the Detection Threshold

SNR $= S/N$ at the detector input. Raise the threshold:

- P_{fa} goes **down** — fewer noise spikes survive
- P_d also goes **down** — some real targets get rejected too

There is no threshold that gives high P_d and low P_{fa} without enough SNR. Detection is an **SNR business.**

Common operating point: $P_d = 0.9$, $P_{fa} = 10^{-6}$ requires SNR ≈ 13 dB on a steady target.

The Receiver Operating Characteristic (ROC)

Plot P_d vs P_{fa} as the threshold sweeps. Each **SNR** value gives a different curve.

- Higher SNR pushes the curve toward the top-left corner (perfect detector)
- Pick an operating P_{fa}, read off the achievable P_d
- Or fix a mission P_d and read off the required SNR

The ROC turns a vague ”good detector” statement into a single requirement: **SNR at the detector input.**

Integration Gain: Spending Time for SNR

Combine N pulses on the same target. SNR improves:

- **Coherent integration** — sum complex returns. Gain $= N$ linear
  - $+10\log_{10}(N)$ dB — 100 pulses buy 20 dB
- **Non-coherent integration** — sum magnitudes. Gain $\approx \sqrt{N}$
  - $\approx +5\log_{10}(N)$ dB — 100 pulses buy $\sim$10 dB

Coherent integration requires phase coherence across the dwell. Modern AESAs lean on this hard.

Integration buys SNR — coherently with N, non-coherently with $\sqrt{N}$.

Closing the Loop on $R_{\max}$

Recall from L3:

$$R_{\max} = \left[\frac{P_t \, G_t \, G_r \, \lambda^2 \, \sigma}{(4\pi)^3 \, S_{\min}} \right]^{1/4}$$

We treated $S_{\min}$ as a constant. It is actually:

$$\boxed{S_{\min} = \frac{kTB \cdot \mathrm{NF} \cdot \mathrm{SNR}_{\mathrm{req}}}{G_{\mathrm{int}}}}$$

- Tighter mission $P_d \Rightarrow$ higher $\mathrm{SNR}_{\mathrm{req}} \Rightarrow$ larger $S_{\min} \Rightarrow$ shorter $R_{\max}$
- More integration $\Rightarrow$ smaller $S_{\min} \Rightarrow$ longer $R_{\max}$
- This is how a ”longer dwell” becomes a ”longer range”

Today's Interactive Demo: ROC Explorer

Today's interactive demo.

- Drag the threshold and watch the operating point slide along one ROC curve
- Twin panels: the noise and signal+noise densities move with it
- The shaded tails *are* P_{fa} and P_d — read them off directly
- Raise SNR, or pick N pulses, to jump to a better curve entirely

Companion script: `L8_PdPfaAndROC.m`

L9 is a Project 1 work day — we put L1–L8 to use.

Quick Exercise / Type-along: Set the Threshold

Type this with me. Bare MATLAB — no toolboxes.

```
rng(495); M = 1e6;                    % a million noise-only cells
Pfa_want = 1e-3;                      % the false-alarm contract
T = sqrt(2)*erfcinv(2*Pfa_want);      % threshold that honors it
fprintf('T = %.3f  Pfa = %.2e\n', T, mean(randn(M,1) > T));
for snr_dB = [6 10 13]
    d = sqrt(2*10^(snr_dB/10));       % separation, in noise sigmas
    Pd_count = mean((d + randn(M,1)) > T);
    Pd_calc  = 0.5*erfc((T-d)/sqrt(2));
    fprintf('%2d dB: Pd = %.3f (calc %.3f)\n', snr_dB, Pd_count, Pd_calc);
end
```

1. T never moved. P_{fa} never moved. Only P_d moved. As the B-21, which number can you attack?
2. Why contract for $P_{fa} = 10^{-3}$ and not the 10^{-6} from the reading?

Wrap-Up

Key ideas:

- Detection is a thresholded coin flip set by SNR
- ROC curves trade P_d for P_{fa} at fixed SNR
- Coherent integration: $+10 \log N$ dB; non-coherent: $+5 \log N$ dB
- $S_{\min}$ in the radar range equation is a P_d/P_{fa} contract, not a constant

Next lesson:

- **L9: Project 1 work day** — B-21 threat detection range analysis
- HW1 due at the start of L10