ECE 495 – Electromagnetic Warfare and Survivability

Block 2: RF Electronic Warfare
Lesson 13: Angle of Arrival

Juan Jurado, Col, USAF, Ph.D.
Permanent Professor and Head
Electrical and Computer Engineering

Fall 2026

Where We Left Off

L12 built the RWR's PDW: frequency, pulse width, TOA, power — and one field we measured but never explained. **AoA**.

AoA is the only *spatial* parameter in the word. It is how the processor tells two emitters apart when their pulses overlap, and how the cockpit knows which way to look.

Today: how an antenna with no moving parts answers *where is it?*

Lesson Objectives

By the end of this lesson, you will be able to:

- Explain why **AoA** is the hardest field in the PDW to measure well
- Describe **amplitude comparison** and where its accuracy runs out
- Derive the **interferometer** relation $\Delta\phi = \frac{2\pi d}{\lambda} \sin\theta$
- Trace the **accuracy–ambiguity trade** that sets baseline length

Why AoA Is the Hard Parameter

Frequency, PW, and PRI are all *timing* measurements on a single antenna. AoA is different — it is geometry, and the RWR is on a moving aircraft.

- The field the processor **deinterleaves** on — it splits emitters that share a band
- It cues the aircrew and points countermeasures at the right threat
- One bearing is a **line**, not a fix — a position needs two (L14)

Every other PDW field says what. AoA says where.

Two Ways to Find the Angle

A passive receiver has two physical handles on bearing:

- **Amplitude** — how *loud* the signal is in beams that point different ways
- **Phase** — how the *wavefront* arrives at antennas spaced apart

Method	Reads	Character
Amplitude comparison	Power ratio across beams	Coarse, cheap, instant
Phase interferometry	Phase across a baseline	Fine, but can wrap

Most RWRs do amplitude comparison; precision systems add interferometry.

Amplitude Comparison: Squinted Beams

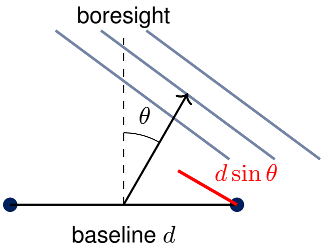
Point several broad antennas in different directions so their patterns overlap. A signal off boresight is *louder* in the beam aimed nearer to it.

- The classic RWR: **four antennas, one per quadrant**; the power ratio between adjacent beams gives bearing within the overlap
- **Single pulse** — no waiting, no second emitter needed
- Wideband and cheap; degrades gracefully as signals weaken

Accuracy is limited by how well you *know* each antenna pattern. Typical four-quadrant bearing lands within roughly 10–30° — enough to label a quadrant, not to target.

Phase Interferometry: The Geometry

Two antennas, baseline d apart. A plane wave from off-boresight angle θ reaches the far element after traveling an extra $d \sin\theta$.



That extra path is a measurable **phase difference** between the two channels.

From Phase to Angle

The extra path $d \sin\theta$ becomes phase by dividing by wavelength:

$$\Delta\phi = \frac{2\pi d}{\lambda}\,\sin\theta \qquad \Longrightarrow \qquad \theta = \arcsin\left(\frac{\lambda\,\Delta\phi}{2\pi d}\right)$$

- Measure one number, $\Delta\phi$; invert for the bearing θ
- A fixed phase error maps to a *smaller* angle error as d/λ grows
- Long baselines give **sub-degree** accuracy — far past amplitude comparison

Stretch the baseline, sharpen the bearing.

UNITED STATES AIR FORCE ACADEMY

The Catch: Phase Wraps

A phase meter cannot tell $\Delta\phi$ from $\Delta\phi + 2\pi$. The reading folds into $[-\pi, \pi]$ — so several angles share one measurement.

- Unambiguous across $\pm 90^\circ$ requires $|\Delta\phi| \leq \pi$, i.e. $\boxed{d \leq \lambda/2}$
- But $d \leq \lambda/2$ is a *short* baseline — back to coarse accuracy
- Make d large and the phase wraps — a fine but *wrong* bearing

Accuracy wants a long baseline; unambiguity wants a short one. You cannot have both with a single pair.

Resolving It: Multi-Baseline

Use more than two elements — a **short** baseline and a **long** one:

1. The short pair ($d \leq \lambda/2$) gives a **coarse, unambiguous** bearing
2. That coarse bearing predicts which 2π cycle the long pair is in
3. Unwrap the long pair's phase with that cycle, then invert for a **fine** bearing

- Short pair picks the cycle; long pair sets the precision
- Real systems use several baselines and calibrate each across the band

Amplitude vs Interferometry

	Amplitude comparison	Interferometry
Reads	Power ratio across beams	Phase across a baseline
Accuracy	Coarse (~ 10–30°)	Fine ($< 1^\circ$ possible)
Pulses	Single pulse	Single pulse
Cost / size	Low; a few antennas	Higher; spaced array + cal
Failure mode	Pattern calibration error	Phase wrap / ambiguity

- Many fielded RWRs amplitude-compare for the quadrant, then interferometer-refine when precision matters
- Both are **passive** and **instantaneous** — no scan, no transmit

Today's Interactive Demo: Phase to Bearing

Today's interactive demo.

- Turn a true AoA into the phase across a short and a long baseline
- Watch the long baseline wrap — the naive inversion lands on the wrong angle
- Use the short baseline to unwrap the long one and recover a fine bearing

Companion script: `L13_InterferometricAoA.m`

In L14 we take these bearings to two receivers and cross them into a fix.

Quick Exercise / Type-along: Pick the Baseline

A 10 GHz threat ($\lambda \approx 3$ cm). Reason about each choice:

1. You set $d = 1.5$ cm. Ambiguous or not? What does that cost you?
2. You set $d = 15$ cm. What do you gain, and what breaks?
3. Both elements drift 20° in measured phase. Which baseline's bearing moves more?
4. Why does amplitude comparison never have this ambiguity problem?

Wrap-Up

Key ideas:

- AoA is the only spatial PDW field — it deinterleaves, cues, and warns
- Amplitude comparison: power ratio across beams — coarse, cheap, instant
- Interferometry: $\Delta\phi = \frac{2\pi d}{\lambda} \sin\theta$ — fine bearing from one phase measurement
- Long baseline sharpens accuracy but wraps; multi-baseline buys both

Next lesson:

- **L14: Triangulation and fusion** — crossing bearings into a fix
- Pre-flight quiz for L14 due before class