

ECE 215/S – Principles of Electronic Cyber Warfare

Block 3: Communication Systems, Radar, and Electronic Warfare

Lesson 28: Modulation and Amplitude Modulation

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Objective 3.1

I can determine the modulation index, output signal, and output signal bandwidth of an Amplitude Modulation (AM) system and assess whether the system is under-modulated, over-modulated, or fully modulated.

- This objective spans two lessons — today the fundamentals, then it **continues at Lesson 29**
- Assessed on **Quiz 8** (Objectives 3.1–3.2), due at **Lesson 32**

Where We Are Going

By the end of this lesson, you will be able to:

- Explain **why** we modulate rather than transmitting a message directly
- Name the three properties of a carrier that can be modulated: **AM, FM, PM**
- Write the output of a **function multiplier** — every term, with its frequency and its amplitude
- Compute the **modulation index** α, classify the signal, and draw its **spectrum**

Block 3 begins here. Project 2 built the *decision* half of a fire-control system. Before a seeker can compare signal against noise, it has to **put the signal on a carrier and get it back** — Lessons 28–30. Today is the "put it on" half, and it is built from three things you already own: Lesson 6's sinusoid, Lesson 15's spectrum, and Lesson 21's bias B.

Why Modulate?

Modulation is varying a property of one signal (the *carrier*) according to another (the *message*).

- **Pick your transmission frequency.** A 3 kHz voice signal is stuck at 3 kHz. Modulated onto a 100 MHz carrier, it goes where you choose
- **Smaller antennas.** Antenna size scales with wavelength (Lesson 31). A quarter-wave antenna at 3 kHz would be 25 km long; at 100 MHz it is 75 cm
- **Many channels in one band.** Put different messages on different carriers and they coexist — that is frequency division multiplexing
- **Long-distance transmission** becomes practical

Modulation is what makes wireless communication physically possible.

Three Things You Can Modulate

A carrier is Lesson 6's sinusoid with no bias: $v_c(t) = A_c \cos(2\pi f_c t + \phi)$. It has exactly three properties to vary:

- **Amplitude Modulation (AM)** — vary A_c
  - Cheap, readily available technology; long-distance; uses less bandwidth
  - Vulnerable to noise, because noise is also amplitude
- **Frequency Modulation (FM)** — vary f_c
  - Noise resistant, higher quality; costs more bandwidth
- **Phase Modulation (PM)** — vary ϕ
  - Noise resistant; the basis of most digital modulation, including cell phones

This course focuses on **AM**, because it is the one you can work by hand — and the one radar and jamming build on.

Message and Carrier

Message signal $v_m(t)$

- Contains the data you want to send
- Can be any format — analog or digital
- Relatively **low** frequency
- It does the modulat**ing**

Carrier signal $v_c(t)$

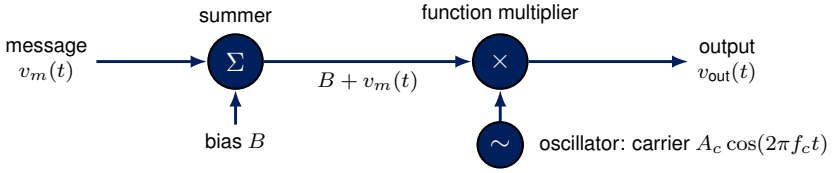
- "Carries" the message
- **Always** a sinusoid
- Much **higher** frequency than the message
- It is the thing being modulat**ed**

The message modulates. The carrier is modulated. Keep the roles straight.

From here on f_c is the **carrier** frequency. A filter cutoff — Lesson 15's f_c — is written f_{cutoff}, as on the equation sheet.

The Function Multiplier

A **function multiplier**, or **mixer**, multiplies a biased message by a carrier:



$$v_{\text{out}}(t) = \big[\, B + v_m(t) \,\big] \cdot A_c \cos(2\pi f_c t)$$

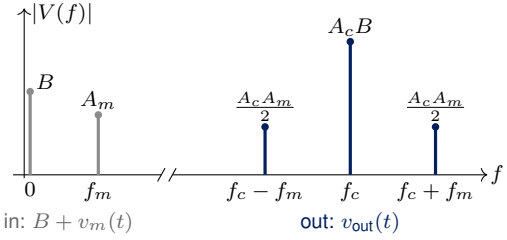
- B is a **DC bias** added before multiplication — Lesson 21's $v_{\text{out}} = K\, v_{\text{in}} + B$, here with $K = 1$
- The output sits at the **carrier frequency**; its amplitude rises and falls with the message
- That outline is the **envelope**, $A_c\big[B + v_m(t)\big]$ — the message, riding on the carrier

The AM Equation

For a single tone $v_m = A_m \cos(2\pi f_m t)$, the biased product expands to **three terms**:

$$v_{\text{out}}(t) = \underbrace{\frac{A_c A_m}{2} \cos\big[2\pi(f_c - f_m)t\big]}_{\text{lower sideband}} + \underbrace{A_c B \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{\frac{A_c A_m}{2} \cos\big[2\pi(f_c + f_m)t\big]}_{\text{upper sideband}}$$

- $\cos a \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$: a **difference and sum** pair — the **lower** and **upper sidebands**
- With no bias there is **no carrier term**. A bias B puts it back as $A_c B \cos(2\pi f_c t)$
- On Lesson 15's spectrum the message spike at f_m becomes two spikes of height $A_c A_m/2$ at $f_c \pm f_m$, and the bias spike moves from $f = 0$ to f_c with height $A_c B$



Multiplying in time becomes shifting in frequency.

Modulation Index

$$\alpha = \frac{\max(\textbf{message})}{B}$$

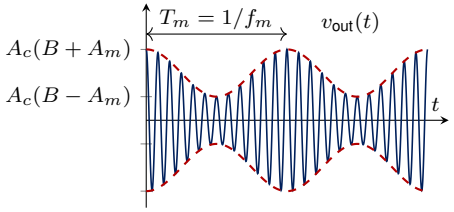
- α compares how far the envelope swings against the bias holding it up
- For a single tone, $\max(\text{message}) = A_m$, so $\alpha = A_m/B$
- The carrier amplitude A_c multiplies *both* the swing and the bias, so it **cancels** — α does not depend on it
- It is a **ratio** — no units

- $\alpha < 1$ — **under-modulated**
- $\alpha = 1$ — **fully modulated** (100%)
- $\alpha > 1$ — **over-modulated**
- On a single-tone **spectrum**, α is twice a sideband's height over the carrier's — the sheet's $2 \cdot Amp_{SB}/Amp_{Carrier}$
- It decides which demodulator will work (Lesson 30)

Method: Analyzing an AM Signal

Given a message, a bias B, and a carrier:

1. **Modulation index:** $\alpha = \max(\text{message})/B$; for a single tone, A_m/B
2. **Classify:** $\alpha < 1$ under-, $\alpha = 1$ fully, $\alpha > 1$ over-modulated
3. **Output terms:** sidebands $A_c A_m/2$ at $f_c \pm f_m$; carrier $A_c B$ at f_c — none if $B = 0$
4. **Spectrum:** one labeled stem per term. **Bandwidth:** $BW = f_{high} - f_{low} = 2f_m$
5. **Check:** $2\times$ (the sideband heights on one side, summed) $\div$ carrier height returns step 1's α



To sketch the waveform: draw the envelope $A_c\big[B + v_m(t)\big]$ first, mirror it below the axis, then fill in the carrier. Label the envelope's maximum and minimum and the *message* period.

For intuition: sketching an AM waveform is not assessed.

Worked Example

A message and a carrier go into a function multiplier:

$$v_m = 3\cos(2\pi \cdot 5t)\,\text{V} \qquad \text{and} \qquad v_c = 4\cos(2\pi \cdot 120t)\,\text{V}$$

Run the Method twice:

(a) with **no bias**, $B = 0$

(b) with a bias of $B = 4\,\text{V}$

For each case, find:

1. α, and whether the signal is under-, fully, or over-modulated
2. Every output term — frequency and amplitude
3. The spectrum and the bandwidth

Work it before the next slide.

Worked Example — Solution

(a) No bias, $B = 0$

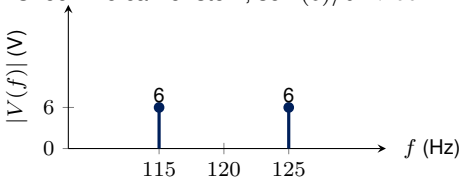
1. $\alpha = 3/0 \to \infty$ **2. Over-modulated**

3. Sidebands $\frac{(4)(3)}{2} = 6$ V at $120 - 5 = 115$ Hz and $120 + 5 = 125$ Hz; **no carrier**, $A_c B = 0$

$v_{\text{out}} = 6\cos(2\pi \cdot 115t) + 6\cos(2\pi \cdot 125t)$ V

4. $BW = 125 - 115 = 10$ Hz $= 2f_m$

5. Check: no carrier stem, so $2(6)/0 \to \infty$



(b) Bias $B = 4$ V

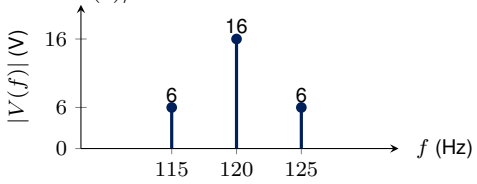
1. $\alpha = 3/4 = 0.75$ **2. Under-modulated**

3. Add the carrier: $A_c B = (4)(4) = 16$ V at 120 Hz

$v_{\text{out}} = 6\cos(2\pi \cdot 115t) + 16\cos(2\pi \cdot 120t)$
$+ 6\cos(2\pi \cdot 125t)$ V

4. $BW = 10$ Hz still: the carrier is *inside* the band

5. Check: $2(6)/16 = 0.75$



Your Turn

A message $v_m = 2\cos(2\pi \cdot 1000t)$ V is biased by $B = 4$ V and multiplied by a carrier $v_c = 5\cos(2\pi \cdot 50{,}000t)$ V.

1. What is the modulation index α?
2. Is the signal under-, fully, or over-modulated?
3. Write every output term — its frequency **and** its amplitude — and draw the spectrum
4. What is the bandwidth?
5. What are the largest and smallest values of the envelope?

Wrap-Up

Key ideas:

- We modulate to **choose a frequency**, **shrink antennas**, and **share a band**
- A carrier has three modulatable properties: **amplitude, frequency, phase**
- A biased message times a carrier gives **three terms**: sidebands of $A_c A_m/2$ at $f_c \pm f_m$ and a carrier of $A_c B$ at f_c
- $\alpha = \max(\text{message})/B$ decides under-, fully, or over-modulated — and that decides your demodulator

Next lesson:

- **Lesson 29 — Objective 3.1 continues**: what each α does to the waveform, bandwidth, and FDM
- **Project 2** is due at Lesson 29 — exact deadline on Gradescope