

ECE 215/S – Principles of Electronic Cyber Warfare

Block 2: Signal Processing

Lesson 15: Frequency Domain and Ideal Filters

Department of Electrical and Computer Engineering
United States Air Force Academy

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Objectives 2.1 and 2.2

I can graph a given sinusoidal signal, including DC components, in both the time and frequency domains and determine its bandwidth.

I can identify types of ideal filters, determine their cutoff frequencies, and analyze their output given an input sinusoidal signal or signal spectrum.

- Two objectives today — this is the densest lesson in the course
- Both assessed on **Quiz 4**

Where We Are Going

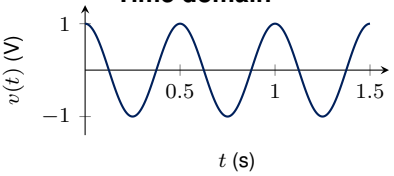
By the end of this lesson, you will be able to:

- Draw a signal in both the **time domain** and the **frequency domain**
- Place a **DC component** correctly in a spectrum
- Determine a signal's **bandwidth**
- Identify the **four ideal filter types** and predict the output spectrum of each

Block 2 begins here. Project 1 is closed out. Block 1's circuit analysis does not go away, but from today it is a *tool* rather than the subject — the subject is now the **signal**.

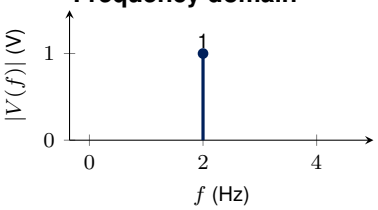
Two Views of the Same Signal

Time domain



Amplitude versus *time*. What an oscilloscope shows — here $T = 0.5$ s, so $f = 1/T = 2$ Hz.

Frequency domain



Amplitude versus *frequency*. What a spectrum analyzer shows — one spike, at 2 Hz and 1 V.

One sinusoid in time is one spike in frequency. Same signal, two views.

Reading a Spectrum

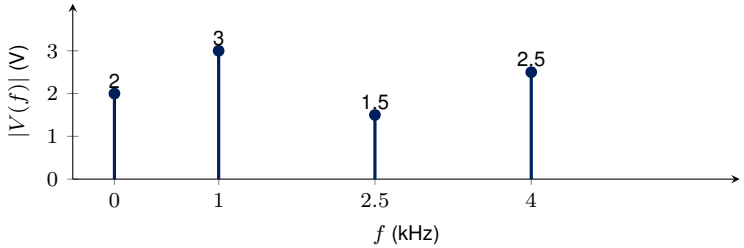
For $v(t) = V_{\text{DC}} + A\cos(2\pi f_0 t + \phi)$, where V_{DC} is Lesson 6's V_{bias} under a new name:

- The **DC term** appears as a spike at $f = 0$, with height V_{DC}
- The **sinusoid** appears as a spike at $f = f_0$, with height A. A *negative* coefficient plots as a positive spike — the height is the **magnitude** of the amplitude
- **Phase does not appear** on an amplitude spectrum — two signals differing only in ϕ look identical here
- We draw the spectrum **one-sided**: only $f \geq 0$, and a spike carries the full amplitude A, not $A/2$. Name the spectrum $V(f)$, so $|V(f)|$ is the height of each spike

Every spike is one sinusoid. Its position is frequency, its height is amplitude.

Multiple Frequencies

A real signal is a **sum** of sinusoids, so its spectrum is a set of spikes.



- This signal has a 2 V DC offset and tones at 1, 2.5, and 4 kHz
- Write it back in time as $v(t) = 2 + 3\cos(2\pi \cdot 1000t) + 1.5\cos(2\pi \cdot 2500t) + 2.5\cos(2\pi \cdot 4000t)$

Bandwidth

$$BW = f_{high} - f_{low}$$

- The width of the band of frequencies the signal actually occupies
- For the previous slide: $f_{high} = 4$ kHz, $f_{low} = 0$ (the DC term), so $BW = 4$ kHz
- **Careful:** if the signal has a DC component, $f_{low} = 0$
- Bandwidth drives almost everything downstream — sampling rate (Objective 2.5), noise (3.8), and how many channels fit in a band (3.1)

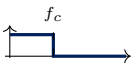
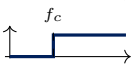
Why Filter?

- **Keep what you want, reject what you do not.** A radio receiver must pass one station and reject every other
- **Remove noise** outside the signal's band — and less bandwidth means less noise (Objective 3.8)
- **Prevent aliasing** before an ADC (Objective 2.6)
- **Split a signal** into bands for different purposes — the crossover in a speaker sends bass to the woofer and treble to the tweeter

A filter is a rule about which frequencies survive.

The Four Ideal Filter Types

A filter is defined by its **cutoff frequency** f_{cutoff} — written f_c for short, and f_{c1}, f_{c2} when there are two.

Type	Passes	Ideal response
Low-pass (LPF)	Everything *below* f_c	
High-pass (HPF)	Everything *above* f_c	
Band-pass (BPF)	Everything *between* f_{c1} and f_{c2}	
Band-stop (BSF)	Everything *except* f_{c1} to f_{c2}	

Ideal means gain exactly 1 in the passband, exactly 0 in the stopband, and an instant transition. No real filter does this (Objective 2.4, Lesson 17). A band-stop filter is also called a **band-reject** filter (BRF), and tuned narrowly it is a **notch** filter. A cutoff is **inclusive**: a spike exactly *at* f_c passes ($f \leq f_c$ low-pass, $f \geq f_c$ high-pass); a band-stop removes only $f_{c1} < f < f_{c2}$.

Method: Finding an Output Spectrum

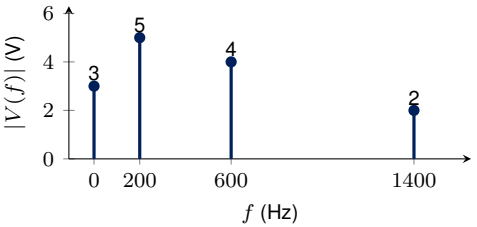
To find the output spectrum, go spike by spike:

1. Draw the **input spectrum** — one stem per sinusoid, height labeled, DC at $f = 0$
2. Draw the **filter response** on the same frequency axis, as a dashed wall at each cutoff
3. For each spike, ask: is it in the passband or the stopband?
4. **Passband** $\rightarrow$ it survives at full height. **Stopband** $\rightarrow$ it disappears
5. Write the surviving spikes back as a time-domain signal
6. **Check:** every survivor kept its input height, and the bandwidth never grew

Ideal filters do not attenuate:
each spike survives untouched, or vanishes.

Worked Example

$$v_{\text{in}}(t) = 3 + 5\cos(2\pi \cdot 200t) + 4\cos(2\pi \cdot 600t) + 2\cos(2\pi \cdot 1400t)$$



State the input **bandwidth**, then **draw the output spectrum** for each filter:

1. LPF, $f_c = 400$ Hz
2. HPF, $f_c = 400$ Hz
3. BSF, $f_{c1} = 400$ Hz, $f_{c2} = 1$ kHz
4. BPF, $f_{c1} = 700$ Hz, $f_{c2} = 1.2$ kHz

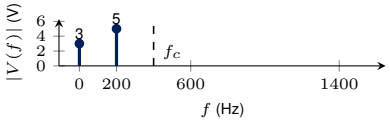
Work it before the next slide. The input spikes sit at 0, 200, 600, and 1400 Hz.

Worked Example — Solution

$f_{low} = 0$ (DC is present) and $f_{high} = 1400$ Hz, so $BW = 1.4$ kHz. Survivors keep their input heights.

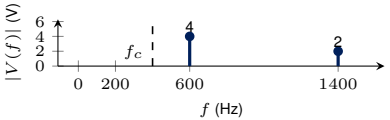
1. LPF, $f_c = 400$ Hz

$v_{\text{out}} = 3 + 5\cos(2\pi \cdot 200t)$



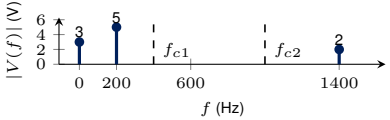
2. HPF, $f_c = 400$ Hz — DC is gone

$v_{\text{out}} = 4\cos(2\pi \cdot 600t) + 2\cos(2\pi \cdot 1400t)$



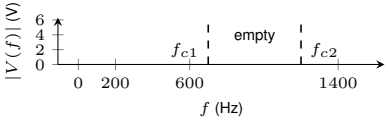
3. BSF, 400 Hz–1 kHz

$v_{\text{out}} = 3 + 5\cos(2\pi \cdot 200t) + 2\cos(2\pi \cdot 1400t)$



4. BPF, 700 Hz–1.2 kHz

nothing lies in the band, so $v_{\text{out}}(t) = 0$



An empty output is a real answer — write $v_{\text{out}}(t) = 0$ and say why. And an HPF always removes DC — exactly how the demodulators in Lesson 30 strip a bias.

Your Turn

$$v_{\text{in}}(t) = 2 + 4\cos(2\pi \cdot 100t) + 7\cos(2\pi \cdot 1200t) + 3\cos(2\pi \cdot 6000t)$$

1. Sketch the input spectrum — label every position and height — and state the bandwidth
2. Find the output of an LPF with $f_c = 2$ kHz
3. Draw the output of a BSF blocking 500 Hz to 2 kHz
4. What single filter would leave only the 1200 Hz tone?

Wrap-Up

Key ideas:

- Time and frequency are **two views of one signal** — one sinusoid is one spike
- **DC sits at** $f = 0$, and it counts: $BW = f_{high} - f_{low}$, so a DC term pins $f_{low} = 0$
- Four ideal types: **LPF, HPF, BPF, BSF** — band-stop is also called **band-reject (BRF)**
- Each spike either **survives untouched or vanishes**. Ideal filters do not attenuate

Next lesson:

- **Lesson 16 — Objective 2.3: Complex Math and Impedance.** Drawing a spectrum threw **phase** away; Lesson 16 gives you the object that carries it back
- **Project 1 Submissions 2–3** were due **Wed 16 Sep, 2359**
- **Quiz 3** (Objectives 1.6–1.7) is due **Mon 21 Sep, 2359**
- **Quiz 4** (Objectives 2.1–2.2) is due at **Lesson 18** (28/29 Sep)