1. Two planes are separated by 500 m. If your RADAR has a pulse width of 1 µs, can you see both planes?

$\Delta R =$ 150m, Yes

$\Delta R = \dfrac{c\tau}{2} = \dfrac{(3\times10^8)(1\times10^{-6})}{2} = 150$ m

The planes are 500 m apart, more than ΔR: yes, two separate returns

2. Two planes are separated by 500 m. If your RADAR has a pulse width of 10 µs, can you see both planes?

$\Delta R =$ 1.5km, No

$\Delta R = \dfrac{c\tau}{2} = \dfrac{(3\times10^8)(10\times10^{-6})}{2} = 1{,}500$ m $= 1.5$ km

The planes are 500 m apart, less than ΔR: no, the two returns merge into one

3. A RADAR that has a PRF of 10kHz is trying to detect an object 25 km away. Will it be able to unambiguously detect the range?

$R_{unamb} =$ 15km, No

$PRI = 1/PRF = 1/(10\times10^3 \text{ Hz}) = 100\ \mu$s

$R_{\text{unamb}} = \dfrac{c\cdot PRI}{2} = \dfrac{(3\times10^8)(100\times10^{-6})}{2} = 15$ km

25 km $>$ 15 km: no. The echo returns after the next pulse has left, and the RADAR reports $25-15=10$ km.

4. An aircraft is approaching a RADAR using an approach angle of 20°. The aircraft is traveling at a speed of 200 mph. The RADAR emits its signal at a frequency of 200 MHz. What is the frequency of the return signal?

$f =$ 200.0001121 MHz

$v = 200$ mi/h $\times$ 1.61 km/mi $= 322$ km/h; $\div 3.6 = 89.44$ m/s

$\Delta f = \dfrac{2v\cos\theta}{c} f_{\text{original}} = \dfrac{2(89.44)\cos 20^\circ}{3\times10^8}(200\times10^6) = 112.1$ Hz

Approaching is closing, so the shift is added:

$f_{\text{return}} = 200$ MHz $+ 112.1$ Hz $= 200.0001121$ MHz

5. An airplane is flying overhead with an approach angle of 60°. If a RADAR transmits a signal at 300 MHz and it returns at 300.000125 MHz, how fast is the airplane traveling?

$v =$ 125 m/s, closing

$\Delta f = 300.000125 - 300 = 0.000125$ MHz $= 125$ Hz. The return is above the transmitted frequency, so the airplane is closing.

$v = \dfrac{c\Delta f}{2f_{\text{original}}\cos\theta} = \dfrac{(3\times10^8)(125)}{2(300\times10^6)\cos 60^\circ} = 125$ m/s

6. A squadron of planes is approaching a SAM site's RADAR installation. The RADAR has a PRF of 6 kHz and a pulse width of 300 ns.

   (a) What is the PRI? $PRI =$ 166.7μs

   $$PRI = \frac{1}{PRF} = \frac{1}{6 \times 10^3 \text{ Hz}} = 1.667 \times 10^{-4} \text{ s}$$
   $$= 166.7 \ \mu\text{s}$$

   (b) If you are 28 km from the SAM site, does it know where you are (without additional processing)? $R_{unamb} =$ 25 km, No

   $$R_{\text{unamb}} = \frac{c \cdot PRI}{2} = \frac{(3 \times 10^8)(166.7 \times 10^{-6})}{2}$$
   $= 25$ km, and 28 km $>$ 25 km: no

   Your echo returns after the next pulse has left, so the SAM site reports a ghost at $28 - 25 = 3$ km.

   (c) If you are ingressing as a two-ship with 10 m spacing, will the SAM be able to tell there are two of you? $\Delta R =$ 45 m, No

   $$\Delta R = \frac{c\,\tau}{2} = \frac{(3 \times 10^8)(300 \times 10^{-9})}{2} = 45 \text{ m}$$
   Your 10 m spacing is less than ΔR: no, the two-ship looks like one target

7. A squadron of planes is approaching a RADAR installation. The squadron has the following characteristics:

$\sigma = 6\text{m}^2$
$\text{G}_\text{R} = 3.2$
$\text{P}_{\text{r,min, RWR}} = 2\mu\text{W}$
altitude = 1000ft, AGL

The RADAR installation has the following characteristics:

$f = 1\text{GHz}$
$\text{G}_\text{T} = 300$
$\text{P}_\text{T} = 400\text{kW}$
$\text{P}_{\text{r,min,RADAR}} = 300\text{fW}$
altitude = 300ft, AGL

(a) For the RADAR installation, what is the PRF required to unambiguously detect the squadron (consider the lead aircraft) out to 180km?

PRF = 833 Hz

Need $R_{\text{unamb}} = \dfrac{c \cdot PRI}{2} = 180$ km:
$PRI = \dfrac{2(180{,}000\text{ m})}{3 \times 10^8\text{ m/s}} = 1.2$ ms
$PRF = 1/PRI = 833$ Hz. This is the highest PRF that works; any higher shortens R_{unamb}.

(b) The incoming squadron will have 350ft between each plane – what is the maximum RADAR pulse width required to distinguish the planes?

$\tau =$ 712 ns

$\Delta R = 350\text{ ft} \times 0.305\text{ m/ft} = 106.75\text{ m}$
$\Delta R = \dfrac{c\tau}{2}$, so $\tau = \dfrac{2\,\Delta R}{c} = \dfrac{2(106.75)}{3 \times 10^8}$
$= 7.117 \times 10^{-7}\text{ s} = 711.7\text{ ns} \approx 712\text{ ns}$

(c) For the conditions specified, what will be the maximum line of sight distance between the RADAR and the lead aircraft?

$\text{R}_\text{LOS} =$ 111 km

$R_{LOS,mi} = \sqrt{2(1000)} + \sqrt{2(300)}$
$= 44.721 + 24.495 = 69.216$ mi
$69.216\text{ mi} \times 1.61\text{ km/mi} = 111.4\text{ km} \approx 111\text{ km}$

(d) What is the maximum distance from which the RADAR will detect the planes?

R_{RADAR} = **75.6 km**

Two-way, so the radar range equation, with the RADAR's own $P_{R,\min} = 300$ fW. $\lambda = c/f = 0.3$ m

$$R_{\text{radar}} = \left(\frac{P_T G_T^2 \sigma \lambda^2}{(4\pi)^3 P_{R,\min}} \right)^{1/4}$$

$$= \left(\frac{(400{,}000)(300)^2(6)(0.3)^2}{(4\pi)^3(300 \times 10^{-15})} \right)^{1/4}$$

$$= (3.265 \times 10^{19})^{1/4} = 7.56 \times 10^4 \text{ m} = 75.6 \text{ km}$$

(e) What is the maximum distance from which the lead aircraft's RWR will detect the RADAR?

R_{RWR} = **331 km**

One-way, so Friis, with the RWR's $G_R = 3.2$ and its own $P_{R,\min} = 2$ μW:

$$R_{\text{RWR}} = \frac{\lambda}{4\pi} \sqrt{\frac{P_T G_T G_R}{P_{R,\min}}}$$

$$= \frac{0.3}{4\pi} \sqrt{\frac{(400{,}000)(300)(3.2)}{2 \times 10^{-6}}}$$

$$= 3.308 \times 10^5 \text{ m} = 331 \text{ km, by power alone}$$

(f) Who will see who first, and at what range?

Answer = **Planes at 111 km (LOS-limited)**

Nobody detects beyond the 111 km LOS of (c), so take the smaller of each power range and the LOS.

RWR: the smaller of 331 km and 111 km is 111 km (LOS-limited). RADAR: the smaller of 75.6 km and 111 km is 75.6 km.

The longer range detects first: the planes hear the RADAR at 111 km, as soon as they clear the horizon.

(g) If the planes would like to reduce R_{LOS} to 80km, what altitude would they have to drop down to?

Altitude = **317 ft**

80 km ÷ 1.61 km/mi = 49.689 mi. The RADAR stays at 300 ft.

$$\sqrt{2h_{ft}} = 49.689 - \sqrt{2(300)}$$

$$= 49.689 - 24.495 = 25.194$$

$$h_{ft} = (25.194)^2/2 = 317 \text{ ft}$$

(h) The RADAR's return frequency is 1.000000866 GHZ, and the approach angle of the planes is 25°. What is the speed of the planes, in mph.

Speed = 320 mph

Δf = 1.000000866 GHz − 1 GHz = 866 Hz. The return is above the transmitted frequency, so the planes are closing.

$$v = \frac{c\,\Delta f}{2 f_{\text{original}} \cos\theta} = \frac{(3 \times 10^8)(866)}{2(1 \times 10^9)\cos 25^\circ}$$

$= 143.33$ m/s; $\times 3.6 = 516.0$ km/h

516.0 km/h ÷ 1.61 km/mi = 320.5 ≈ 320 mph