

ECE 215/S – Principles of Electronic Cyber Warfare

Block 1: Resistive Circuits and Power Distribution

Lesson 7: Reactive and Apparent Power

Department of Electrical and Computer Engineering
United States Air Force Academy

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Objective 1.4

I can compute the reactive and apparent power values for an AC-powered circuit.

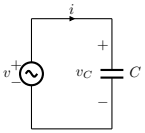
- Lesson 6 assumed purely resistive loads. Real systems have **motors, coils, and cables** — and those are not resistive
- Assessed on **Quiz 2** along with Objectives 1.3 and 1.5

Where We Are Going

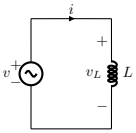
By the end of this lesson, you will be able to:

- Explain how **capacitors** and **inductors** shift the phase between voltage and current
- Distinguish **real power** P, **reactive power** Q, and **apparent power** S, with the right units for each
- Use the **power triangle** and compute **power factor**
- Explain what a low power factor actually costs — the **amperes** it forces, not the VARs themselves

Beyond Resistors: Capacitors and Inductors



i **leads** v by $90°$



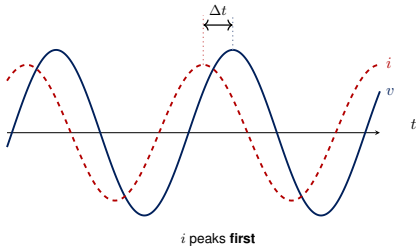
i **lags** v by $90°$

- A **capacitor** stores energy in an *electric* field. Current leads voltage by $90°$
- An **inductor** stores energy in a *magnetic* field. Current lags voltage by $90°$
- Both **store and return** energy each cycle rather than consuming it

Key consequence: they consume **no real power** — but they still draw current, and the wires carrying it are real.

Reactive elements borrow energy and give it back. Resistors keep it.

What Reactance Does to Current



Reactance does more than resize the current — it **shifts it in time**:

$$\phi = \phi_V - \phi_I$$

- i peaks first — i **leads** v: **leading**, capacitive, $\phi < 0$. Mirrored, i **lags**: **lagging**, inductive, $\phi > 0$
- The shifted part of $v\,i$ is **borrowed and returned** each cycle — no net work. That is Q
- Only the in-phase part works: $P = V_{\text{rms}} I_{\text{rms}} \cos\phi$ — all of it at $\phi = 0°$ (resistor), none at $\phi = \pm 90°$ (pure reactance — Lesson 16's subject)

Three Kinds of Power

Everything below uses the same $\phi = \phi_V - \phi_I$:

- **Real power** P — what actually does work and produces heat

$$P = V_{\text{rms}}\, I_{\text{rms}} \cos\phi \qquad \text{watts (W)}$$

- **Reactive power** Q — energy borrowed and returned by the fields, doing no net work

$$Q = V_{\text{rms}}\, I_{\text{rms}} \sin\phi \qquad \text{volt-amperes reactive (VAR)}$$

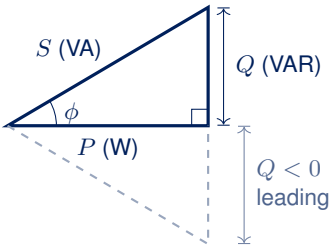
Q is **signed**: lagging (inductive, $\phi > 0$) gives $Q > 0$; leading (capacitive, $\phi < 0$) gives $Q < 0$

- **Apparent power** S — the product the source and wiring must actually supply

$$S = V_{\text{rms}}\, I_{\text{rms}} \qquad \text{volt-amperes (VA)}$$

The units differ on purpose. W, VAR, and VA are not interchangeable.

The Power Triangle



The three combine as a right triangle:

$$S^2 = P^2 + Q^2$$

The angle at the origin is the same ϕ as between voltage and current:

$$\boxed{\text{pf} = \cos\phi = \frac{P}{S}}$$

Power factor runs from 0 to 1. A pf of 1 means every volt-ampere is doing work.
A **leading** load's Q hangs **below** the axis (dashed). Since $\cos\phi > 0$ either way, always say *leading* or *lagging* with a pf.

Why Power Factor Costs Money

Reactive power itself consumes zero net energy — Q flows into the fields and back out each cycle, netting to nothing, and a house meter reads **kWh**, so a homeowner pays *nothing* extra for VARs. The cost is **the amperes**: $I_{\text{rms}} = P/(V_{\text{rms}}\,\text{pf})$, more current for the same watts. Three named harms:

1. I^2R **losses upstream.** Extra amperes burn *real* power as heat in every conductor. Note the **square**: halve the pf, double the current, **quadruple** the loss (Lesson 9's same square law)
2. **Equipment capacity.** Generators, transformers, and cables are thermally *current*-limited — **which is why they are VA-rated**. A pf $= 0.5$ load ties up **twice** the transformer capacity of a unity-pf load at the same P
3. **Voltage regulation.** Reactive current through line *reactance* sags the voltage — the mechanism behind grid **voltage collapse**

Who is billed. Large **industrial and commercial** customers pay **kVA demand charges** or **pf-penalty clauses**, commonly below a pf of about **0.90–0.95**. Residential customers are not.

Correction. A **capacitor bank** *right next to* the inductive load cancels its lagging VARs there, so the reactive current circulates in a **short local loop** rather than crossing the grid. The utility sees pf ≈ 1 and $I_{\text{rms}} \approx P/V_{\text{rms}}$ — the Your Turn does it numerically.

Nobody pays for the VARs. Everybody pays for the amperes the VARs require.

Warning: The Two Mistakes Everyone Makes

1. **Adding P and Q arithmetically.** They are perpendicular. $S^2 = P^2 + Q^2$, so a 3 kW load with 4 kVAR is 5 kVA, not 7

2. **Mixing peak and RMS.** Every equation on these slides takes *RMS* values. If a problem hands you a peak voltage, convert first

Check the units before you check the math.
Wrong units means wrong method.

Method: Solving an AC Power Problem

1. Convert every given value to **RMS**
2. Get the **phase angle** $\phi = \phi_V - \phi_I$, or from the power factor: $\phi = \pm\cos^{-1}(\text{pf})$, taking $+$ for **lagging** and $-$ for **leading**
3. Compute $S = V_{\text{rms}} I_{\text{rms}}$ — this is always the easiest one
4. Get $P = S\cos\phi$ and $Q = S\sin\phi$
5. **Check** $S^2 = P^2 + Q^2$, and label every answer W, VAR, or VA

Rearrangements you will need

$$S = \frac{P}{\text{pf}} \qquad I_{\text{rms}} = \frac{S}{V_{\text{rms}}}$$

$$Q = \pm\sqrt{S^2 - P^2}$$

sign from lagging $(+)$ or leading $(-)$

More than one load on a bus

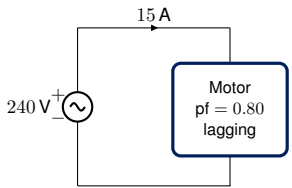
$$P_{\text{bus}} = \sum P \qquad Q_{\text{bus}} = \sum Q \text{ (signed)}$$

$$S_{\text{bus}} = \sqrt{P_{\text{bus}}^2 + Q_{\text{bus}}^2}$$

S does **not** add directly

Worked Example

A motor on a 240 V RMS, 60 Hz line draws 15 A RMS at a lagging power factor of **0.80**.



Find:

1. The apparent power S
2. The real power P
3. The reactive power Q, **with its sign**
4. The phase angle ϕ

"Lagging" means the current lags the voltage — the load is **inductive**, which is what a motor should be.

Work it before the next slide.

Worked Example — Solution

Apparent power

$$S = V_{\text{rms}} I_{\text{rms}} = (240)(15) = 3600 \text{ VA}$$

Real power

$$P = S\cos\phi = 3600(0.80) = 2880 \text{ W}$$

Phase angle

$$\phi = +\cos^{-1}(0.80) = +36.9^\circ \quad \text{(lagging)}$$

Reactive power — get $\sin\phi$ exactly, do not round through the angle:

$$\sin\phi = \sqrt{1 - 0.80^2} = 0.60$$

$$Q = S\sin\phi = 3600(0.60) = +2160 \text{ VAR}$$

Lagging, so Q is **positive**.

Check the triangle

$$\sqrt{2880^2 + 2160^2} = 3600 \checkmark$$

The line carries 3600 VA to deliver 2880 W of useful work.

Your Turn

A 200 V RMS bus feeds **two** loads:

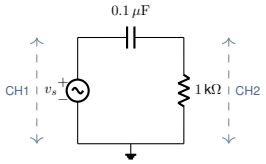
- **Load 1:** 4 kW at a power factor of **0.80 lagging**
- **Load 2:** 4 kW at a power factor of **0.80 leading**

1. Find each load's Q, **with its sign**
2. Find P_{bus} and Q_{bus}
3. Find the bus S and the bus power factor — and confirm the two S values did *not* simply add
4. What current does the bus carry?

Hands-On: Seeing the Phase Shift

Kit: function generator, 2-channel oscilloscope, breadboard, $1\,\text{k}\Omega$ resistor, 0.1 μF capacitor. In **pairs**, about **10 minutes**.

1. Build the **series RC** shown. R **must** sit on the ground side, so both probe grounds share the generator ground
2. Scope **CH1** across the generator, **CH2** across R. v_R is in phase with the current
3. Feed in a **1 kHz sine**. Measure Δt between the two peaks and compute $\phi = (360^\circ/T)\,\Delta t$ — the Lesson 6 formula
4. Compute pf $= \cos\phi$. Which channel peaks first — is the current **leading** or **lagging**?



A *pure* capacitor would shift the current the full 90°. With R in series the shift lands between 0° and 90° — exactly where is **Lesson 16's** business.

Wrap-Up

Key ideas:

- Capacitors and inductors **store and return** energy — they draw current but no real power
- With $\phi = \phi_V - \phi_I$: $P = V_{\text{rms}} I_{\text{rms}} \cos\phi$ (W), $Q = V_{\text{rms}} I_{\text{rms}} \sin\phi$ (VAR), $S = V_{\text{rms}} I_{\text{rms}}$ (VA)
- Right triangle: $S^2 = P^2 + Q^2$ and pf $= \cos\phi = P/S$. Q is **signed**: $+$ lagging, $-$ leading
- On a bus, P and signed Q add. S does **not**
- Nobody pays for the VARs. The cost is the **amperes** they require — I^2R losses, VA-rated capacity tied up, and sagging voltage

Next lesson:

- **Lesson 8 — Objective 1.5: Power and Efficiency.** Efficiency is about **real** power: η divides by P, never by S
- **Quiz 2** (Objectives 1.3–1.5) is due at Lesson 10