ECE215/S – Principles of Electronic Cyber Warfare
Boolean Algebra Quick Reference

Reference only. You will not be asked to simplify a Boolean expression on a quiz or the final exam. Objective 2.7 is gates, truth tables, and SOP and POS expressions.

Notation

Operators	NOT $\overline{A}$ AND $A \cdot B$, or just AB OR $A + B$

This course writes NOT as an overbar, $\overline{A}$. Some other ECE courses write a prime instead, A'. The two mean the same thing.

Order	NOT first, then AND, then OR. Parentheses override. $A + \overline{B}C$ means $A + (\overline{B} \cdot C)$
Long bar	A bar over a group acts like parentheses: work out the group, then invert. $\overline{AB}$ is not $\overline{A}\,\overline{B}$

Identities

Law	Identity	Its dual	What it does
Identity	$A + 0 = A$	$A \cdot 1 = A$	a 0 term or a 1 factor drops out
Null	$A + 1 = 1$	$A \cdot 0 = 0$	a 1 decides a sum, a 0 a product
Idempotent	$A + A = A$	$A \cdot A = A$	repeats drop out, so copy freely
Inverse	$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$	a variable meets its own complement
Double negation	$\overline{\overline{A}} = A$	(its own dual)	two bars cancel
Commutative	$A + B = B + A$	$AB = BA$	order does not matter
Associative	$(A + B) + C = A + (B + C)$	$(AB)C = A(BC)$	grouping does not matter
Distributive	$A(B + C) = AB + AC$	$A + BC = (A + B)(A + C)$	factor out, or multiply out
Absorption	$A + AB = A$	$A(A + B) = A$	the shorter term swallows the longer
Redundant literal	$A + \overline{A}B = A + B$	$A(\overline{A} + B) = AB$	the lone term's complement drops out
Combining	$AB + A\overline{B} = A$	$(A + B)(A + \overline{B}) = A$	**the workhorse** for a truth-table SOP
De Morgan	$\overline{A + B} = \overline{A} \cdot \overline{B}$	$\overline{A \cdot B} = \overline{A} + \overline{B}$	"break the bar, change the sign"

Duals. Swap + with · and 0 with 1, keeping the grouping, and a true identity stays true. **Letters.** Each letter can stand for a variable, a barred variable, or a whole expression. **De Morgan, three variables:** $\overline{A + B + C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$, $\overline{A \cdot B \cdot C} = \overline{A} + \overline{B} + \overline{C}$.
Less common. Consensus: $AB + \overline{A}C + BC = AB + \overline{A}C$; its dual: $(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$.

Reducing an SOP Expression

1. **Clear the long bars.** Break any bar that covers more than one variable (De Morgan), cancel double bars, and multiply out. A truth-table SOP starts at step 2.
2. **Combine.** Two terms that differ in exactly one variable merge, and that variable drops out. Copy a term first if it pairs with more than one neighbor.
3. **Absorb.** Drop a term that contains a shorter term whole. Drop a variable from a term when its complement stands alone as another term.
4. **Factor** what the remaining terms share, to save gates.
5. **Stop** when no rule applies. Two correct reductions can look different.

The check that needs no algebra: two expressions are equal exactly when their truth tables match. Reduction 1, checked row by row:

ABC	000	001	010	011	100	101	110	111
Y	1	1	1	1	0	0	0	1
$\overline{A} + BC$	1	1	1	1	0	0	0	1

Worked Reductions

1. An SOP read off a truth table

$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + ABC$
$= \overline{A}\,\overline{B} + \overline{A}B + ABC$ Combining, twice
$= \overline{A} + ABC$ Combining
$= \overline{A} + BC$ Redundant literal

2. A term used twice (1-rows 011, 110, 111)

$Y = \overline{A}BC + AB\overline{C} + ABC$
$= \overline{A}BC + ABC + AB\overline{C} + ABC$ Idempotent
$= BC + AB$ Combining, twice
$= B(A + C)$ Distributive

3. A long bar (0 only on rows 000, 001)

$Y = \overline{\overline{A}\,\overline{B}} + AC$
$= \overline{\overline{A}} + \overline{\overline{B}} + AC$ De Morgan
$= A + B + AC$ Double negation, twice
$= A + B$ Absorption