

ECE 215/S – Principles of Electronic Cyber Warfare

Block 1: Resistive Circuits and Power Distribution

Lesson 6: AC Circuit Analysis

Department of Electrical and Computer Engineering
United States Air Force Academy

Fall 2026

Objective 1.3

I can calculate the voltages, currents, instantaneous power, and average power associated with devices in a simple AC-powered circuit using tools such as KVL, KCL, voltage and current dividers, Ohm's Law, RMS values, and the power equation.

- Assessed on **Quiz 2** along with Objectives 1.4 and 1.5 **(due Lesson 10)**

Where We Are Going

By the end of this lesson, you will be able to:

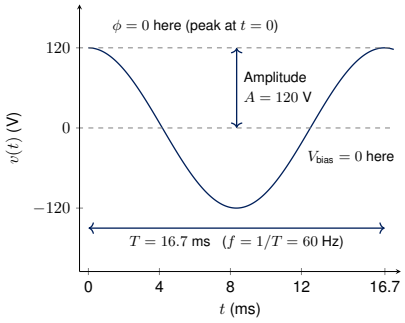
- Read **amplitude**, **frequency**, **period**, **DC bias**, and **phase** off an AC waveform
- Write the equation of a sinusoid from its graph, and sketch the graph from its equation
- Compute the **RMS value** of an AC signal and explain what it means physically
- Find **instantaneous** and **average (real) power** in an AC resistive circuit

Why bother with AC? Spin a coil in a magnetic field and a sinusoid is what comes out:

- **Long-distance transmission** is affordable only because transformers step AC up and back down; transformers do nothing with DC (Lesson 9)
- **The wall outlet is AC** — every device you own converts it back to DC (Lesson 10)

DC is what devices want. AC is how power gets to them.

Anatomy of an AC Signal



$$v(t) = V_{\text{bias}} + A\cos(2\pi f t + \phi)$$

- V_{bias} — DC offset; shifts the whole wave up or down
- A (or V_{pk}) — **amplitude**, the peak strength (peak-to-peak: $V_{\text{pp}} = 2A$)
- f — **frequency** in Hz; $T = 1/f$ is the **period**
- ϕ — **phase shift**, $\phi = \dfrac{2\pi}{T}\,\Delta t$

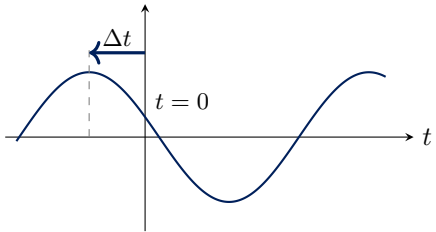
Watch your units and significant figures — most lost points are here.

Phase Angle

Phase tells you *when* the wave starts relative to a plain cosine.

$$\phi = \frac{2\pi}{T}\,\Delta t \qquad \phi^\circ = \frac{360^\circ}{T}\,\Delta t$$

- Measure Δt from the origin **leftward** to the nearest positive peak
- That direction *is* the convention — measuring rightward instead flips the sign of ϕ
- $\phi > 0$ **leads** (peak left of the origin); $\phi < 0$ **lags** (peak right)
- Convert to cosine first and stay there: $\sin(x) = \cos(x - 90^\circ)$



Δt runs **left** from the origin to the nearest positive peak, so ϕ is **positive**: this wave leads.

AC and Ohm's Law: The Good News

For a **resistor**, everything you learned in Lessons 2–4 still applies, moment by moment:

$$v(t) = i(t) \cdot R$$

- Voltage and current through a resistor stay **in phase** — they peak together and cross zero together
- KVL, KCL, and both divider rules work exactly as before
- You can work in peak values or RMS values, as long as you do not mix them — **except power**: the power equations on the next frames require RMS

For resistive AC circuits, nothing about the analysis changes.

What About Power?

Power is still $p(t) = v(t)\, i(t)$ — but both terms are now moving.

For $v(t) = V_{\text{pk}} \cos(2\pi f t)$ across a resistor:

$$p(t) = \frac{V_{\text{pk}}^2}{R} \cos^2(2\pi f t) = \frac{V_{\text{pk}}^2}{2R} \left[1 + \cos(4\pi f t)\right]$$

- **Instantaneous power** oscillates at *twice* the signal frequency, and is never negative for a resistor
- It swings between 0 and V_{pk}^2/R — so quoting the peak overstates what the resistor actually dissipates
- What matters for heating is the **average**: $P_{\text{avg}} = \dfrac{V_{\text{pk}}^2}{2R}$

RMS: The DC Equivalent

The RMS value is the DC voltage that would deliver the same average power.

For *any* sinusoid with no DC bias:

$$V_{\text{rms}} = \frac{V_{\text{pk}}}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_{\text{pk}}}{\sqrt{2}}$$

That is the bias-free case of the general form $V_{\text{rms}} = \sqrt{V_{\text{bias}}^2 + V_{\text{pk}}^2/2}$.

- "Root mean square" — square, average, root
- The $\sqrt{2}$ holds **only** for sinusoids

Why you already know this number.

A US wall outlet is "120 volts" — that is RMS. The actual peak is

$$V_{\text{pk}} = 120\sqrt{2} \approx 170\,\text{V}$$

Every AC rating you have ever seen is an RMS value unless it says otherwise.

Real (Average) Power

Once you are in RMS, the power equation looks exactly like the DC one:

$$P = V_{\text{rms}}\, I_{\text{rms}} = I_{\text{rms}}^2\, R = \frac{V_{\text{rms}}^2}{R}$$

- This is **real power**, measured in watts — the part that actually does work and makes heat
- For a purely resistive circuit, that is the whole story
- Add a capacitor or inductor and it is not — which is Lesson 7

Convert to RMS first, then use the DC power equations unchanged.

Method: AC Circuit Analysis

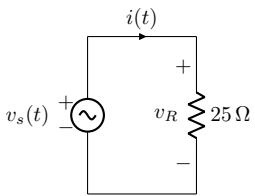
1. **Read the waveform.** Pull off the DC bias V_{bias}, the amplitude A, the frequency f and period $T = 1/f$, and the phase ϕ
2. **Convert to RMS.** No bias: $V_{\text{rms}} = V_{\text{pk}}/\sqrt{2}$. With a bias, use the general form $V_{\text{rms}} = \sqrt{V_{\text{bias}}^2 + V_{\text{pk}}^2/2}$
3. **Apply the DC toolkit** — Ohm's Law, KVL, KCL, the voltage and current dividers — using RMS values throughout
4. **Power in a resistor:** $P = V_{\text{rms}}^2/R$. For a biased signal the DC and AC powers add:

$$P = \frac{V_{\text{bias}}^2}{R} + \frac{\left(V_{\text{pk}}/\sqrt{2}\right)^2}{R}$$

5. **Never mix peak and RMS in one equation.** Pick RMS, convert everything, and stay there

Worked Example

A signal $v_s(t) = 100 + 250\cos(2\pi \cdot 50t)$ volts is applied across a $25\,\Omega$ resistor.



Find:

1. The DC bias and the amplitude
2. The frequency and the period
3. The maximum and minimum values of $v_s(t)$
4. The RMS value of the *AC portion*
5. The **average power** delivered to the $25\,\Omega$ resistor

Work it before the next slide. **Careful:** the DC bias is not part of the $V_{\text{pk}}/\sqrt{2}$ relationship. Handle it separately.

Worked Example — Solution

Read it off the equation

- $V_{\text{bias}} = 100\,\text{V}, \quad A = 250\,\text{V}$
- $f = 50$ Hz, so $T = 1/50 = 20$ ms
- $v_{\max} = 100 + 250 = 350\,\text{V}$
- $v_{\min} = 100 - 250 = -150\,\text{V}$

RMS of the AC portion

$$V_{\text{rms,AC}} = \frac{250}{\sqrt{2}} = 176.8\,\text{V}$$

Average power — two routes, one answer

The resistor responds to *both* parts, and the powers add:

$$P = \frac{100^2}{25} + \frac{\left(250/\sqrt{2}\right)^2}{25} = 400 + 1250$$

$$P = 1650\,\text{W}$$

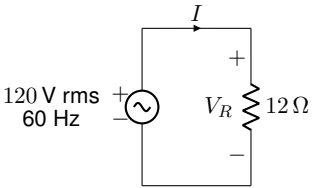
Or collapse the whole signal into one RMS value:

$$V_{\text{rms}} = \sqrt{100^2 + \tfrac{250^2}{2}} = \sqrt{41250} = 203.10\,\text{V}$$

$$P = \frac{V_{\text{rms}}^2}{25} = \frac{41250}{25} = 1650\,\text{W}\ \checkmark$$

Your Turn

A wall outlet supplies $120\,\text{V}$ RMS at 60 Hz to a $12\,\Omega$ heating element.



1. What is the peak voltage across the element?
2. What is the RMS current through it?
3. What is the average power dissipated?
4. What is the *peak instantaneous* power, and why is it not the answer to 3?

Hands-On: RMS on the Bench

Kit: function generator, oscilloscope, DMM. In **pairs**, about **10 minutes**.

1. Set the function generator to a **60 Hz sine**, 10 V peak-to-peak.
2. Set the scope's **volts/div** and **time/div** so one full cycle fills the graticule, then read amplitude and period by counting divisions.
3. On the scope, measure V_{pk} and the period T; confirm $f = 1/T$.
4. Put the DMM on **AC V** across the same signal — that reading is V_{rms}. Compare it to $V_{\text{pk}}/\sqrt{2}$.
5. Change the amplitude and check the ratio again. It is $\sqrt{2}$ *only* because this is a sinusoid.

Wrap-Up

Key ideas:

- An AC signal is $v(t) = V_{\text{bias}} + A\cos(2\pi f t + \phi)$ — read every term off the graph
- For **resistors**, voltage and current stay in phase; all Block 1 tools apply unchanged
- **RMS** is the DC-equivalent value: $V_{\text{rms}} = V_{\text{pk}}/\sqrt{2}$ for a sinusoid
- In RMS terms, $P = V_{\text{rms}} I_{\text{rms}}$ — the familiar DC power equation

Next lesson:

- **Lesson 7 — Objective 1.4: Reactive and Apparent Power.** What happens when voltage and current fall out of phase
- **Quiz 1** (Objectives 1.1–1.2) is due tonight