

ECE 215/S – Principles of Electronic Cyber Warfare

Block 2: Signal Processing

Lesson 16: Complex Math and Impedance

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Objective 2.3

I can model capacitors and inductors as complex resistors and use circuit analysis tools (such as voltage dividers, Ohm's Law, and the power equation) to calculate voltage, current, and power in AC circuits containing these components.

- Lesson 15 **threw phase away** to draw a spectrum. Today we get the object that carries amplitude and phase together
- Lesson 7 said capacitors and inductors shift phase — today we get a **number** for the shift
- Once we do, **every tool from Block 1 works again** — unchanged
- Assessed on **Quiz 5** and the **midterm**

Where We Are Going

By the end of this lesson, you will be able to:

- Convert between **rectangular** and **phasor (polar)** form, by hand and on your calculator
- Write a sinusoid as a **phasor**, and read a phasor back as a sinusoid
- Write the **impedance** of a resistor, capacitor, and inductor
- Apply **voltage dividers and Ohm's Law** to an RLC circuit, in series and in parallel
- Get the **phase angle**, **power factor**, and **real power** from an impedance

Why Complex Numbers?

A sinusoid has two independent properties: **amplitude** and **phase**.

- A single real number cannot carry both
- A **complex number** carries exactly two — magnitude and angle
- So we represent each voltage and current as a complex number, and the circuit algebra from Block 1 goes through untouched
- The cost is arithmetic. The benefit is that **no new circuit theory is needed**

Complex numbers are bookkeeping for phase. Nothing about the circuits changed.

Rectangular and Phasor Form

Rectangular — $Re + jIm$

- Re is the **real** part, Im the **imaginary** part
- Best for **addition and subtraction**

Phasor (polar) — $A\angle\phi$

- A is the magnitude, ϕ is the angle
- Best for **multiplication and division**
- This is the row labeled **Phasor** on the equation sheet. The exponential form $Ae^{j\phi}$ is the same number written a third way

Converting

$$A = \sqrt{Re^2 + Im^2} \qquad \phi = \tan^{-1}\left(\frac{Im}{Re}\right)$$

$$Re = A\cos\phi \qquad Im = A\sin\phi$$

Watch the quadrant. $\tan^{-1}$ cannot tell quadrant II from IV — sketch the point before trusting your calculator.

Add in rectangular. Multiply and divide in polar. Convert freely.

Calculator Skills

Your calculator will do this for you — learn how **before** the quiz.

- The Casio fx-115ES PLUS has a **complex mode**: `MODE 2`, which shows `CMPLX` on the display
- `SHIFT 2 3` converts the answer to phasor form, `SHIFT 2 4` back to rectangular
- Set the angle unit with `SHIFT SETUP 3` — **degrees**, not radians — and check for the `D`
- The Objective 2.3 reading has a step-by-step appendix for this calculator

A calculator error in complex mode looks exactly like a real answer. Check every result.

Impedance

Impedance Z is resistance generalized to include phase. It is measured in ohms.

Element	Impedance	Behavior
Resistor	$Z_R = R$	No phase shift
Capacitor	$Z_C = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C}$	Current *leads* by 90°
Inductor	$Z_L = j\omega L$	Current *lags* by 90°

where $\omega = 2\pi f$.

- **Impedance depends on frequency.** That single fact is what makes filters possible

What Frequency Does to Each Element

Capacitor: $|Z_C| = 1/(\omega C)$

- At **DC** ($\omega \to 0$): $|Z_C| \to \infty$ — an **open circuit**
- At **high** f: $|Z_C| \to 0$ — a **short circuit**

Inductor: $|Z_L| = \omega L$

- At **DC**: $|Z_L| \to 0$ — a **short circuit**
- At **high** f: $|Z_L| \to \infty$ — an **open circuit**

Capacitors block DC and pass high frequencies.
Inductors do the reverse.

Memorize this behavior. It lets you identify a filter type by inspection at Lesson 17.

Sinusoid and Phasor

At a known frequency a sinusoid is just two numbers, so one complex number holds it:

$$v(t) = V_m \cos(2\pi f t + \phi) \qquad \longleftrightarrow \qquad V = V_m \angle \phi$$

- **Going in:** read them off the cosine — $v(t) = 10\cos(2\pi \cdot 1000t + 30°)$ V $\Rightarrow V = 10\angle 30°$ V
- **Coming back:** put them into a cosine at the *same* frequency — $V_C = 8.47\angle -32.1°$ V $\Rightarrow v_C(t) = 8.47\cos(2\pi \cdot 1000t - 32.1°)$ V
- **Amplitude or RMS?** A phasor carries whichever the problem gives you — an **amplitude** from $V_m\cos(\cdot)$, an **RMS** value from V_{rms} — and the answer returns in the same kind; $\sqrt{2}$ only converts between them. **Block 2 examples are amplitudes**
- The reading marks phasors with a tilde, $\widetilde{V}$; here a capital with an angle already is one

Same frequency in, same frequency out.
Only magnitude and phase change.

Method: Analyzing an RLC Circuit

1. Compute $\omega = 2\pi f$ and write each source as a **phasor**
2. Replace every element with its **impedance**: R, $1/j\omega C$, $j\omega L$
3. Combine impedances as you combined resistances in Lesson 4 — **series adds, parallel is product over sum**
4. Apply **Ohm's Law** ($V = IZ$) and the **voltage divider** as usual
5. Convert to phasor form, and write it back as a cosine if $v(t)$ or $i(t)$ was asked for
6. **Check the angle against the circuit:** $\angle Z_{\text{EQ}} < 0$ is capacitive, current **leads**; $\angle Z_{\text{EQ}} > 0$ is inductive, current **lags**

Conversions you will need

$$A = \sqrt{Re^2 + Im^2} \quad \phi = \tan^{-1}\frac{Im}{Re}$$

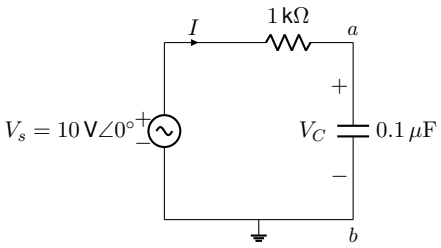
$$Re = A\cos\phi \quad Im = A\sin\phi \quad \frac{1}{j} = -j$$

One Lesson 4 habit does not carry over

$$Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$ still holds

but "smaller than the smallest branch" does **not** — two opposite reactances in parallel can exceed *both*.

Worked Example



$V_s = 10\,\text{V}\angle 0°$ at $f = 1$ kHz — that is, $v_s(t) = 10\cos(2\pi \cdot 1000t)$ V, an **amplitude** phasor.

Find:

1. Z_C and Z_{EQ}
2. V_C in phasor form, then $v_C(t)$
3. The current I, then $i(t)$
4. ϕ, the power factor, and the real power P

Work it before the next slide.

Worked Example — Solution

1. Impedances

$$\omega = 2\pi(1000) = 6283 \text{ rad/s}$$

$$Z_C = \frac{-j}{(6283)(0.1 \times 10^{-6})} = -j1592\,\Omega$$

$$Z_{\text{EQ}} = 1000 - j1592\,\Omega = 1880\angle -57.9^\circ \ \Omega$$

2. Voltage divider — same formula as Lesson 4

$$V_C = V_s \frac{Z_C}{Z_{\text{EQ}}} = 10\angle 0^\circ \cdot \frac{1592\angle -90^\circ}{1880\angle -57.9^\circ}$$

$$V_C = 8.47\angle -32.1^\circ \text{ V} \quad (\text{lagging } V_s)$$

$$v_C(t) = 8.47\cos(2\pi \cdot 1000t - 32.1^\circ) \text{ V}$$

3. Ohm's Law for the current

$$I = \frac{V_s}{Z_{\text{EQ}}} = \frac{10\angle 0^\circ}{1880\angle -57.9^\circ} = 5.32\angle +57.9^\circ \text{ mA}$$

$$i(t) = 5.32\cos(2\pi \cdot 1000t + 57.9^\circ) \text{ mA}$$

4. Angle, power factor, real power

$$\phi = \phi_V - \phi_I = 0^\circ - 57.9^\circ = -57.9^\circ = \angle Z_{\text{EQ}}$$

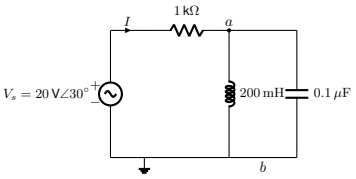
$$\text{pf} = \cos\phi = 0.532, \ \textbf{leading} \text{ (capacitive)}$$

Power needs RMS, so divide each amplitude by $\sqrt{2}$:

$$P = \left(\frac{10}{\sqrt{2}}\right)\left(\frac{5.32\,\text{mA}}{\sqrt{2}}\right)(0.532) = 14.2 \text{ mW}$$

Check: $57.9^\circ + 32.1^\circ = 90.0^\circ$, so the capacitor's current leads its own voltage by exactly 90°. And these are the $1\,\text{k}\Omega$ **and 0.1** μ**F you scoped at Lesson 7**: that shift and pf $= 0.53$ are what you measured.

Adding a Parallel Branch



$v_s(t) = 20\cos(2\pi \cdot 1000t + 30°)$ V, so $V_s = 20\angle 30°$ V at $f = 1$ kHz. Find $i(t)$.

With $\omega = 2\pi(1000) = 6283.2$ rad/s:

$$Z_L = j\omega L = +j1256.6\,\Omega$$

$$Z_C = -j/(\omega C) = -j1591.5\,\Omega$$

$$Z_L Z_C = L/C = 2.000 \times 10^6\ \Omega^2 \text{ (real, exactly)}$$

$$Z_L + Z_C = -j334.9\,\Omega \quad Z_L \parallel Z_C = \frac{Z_L Z_C}{Z_L + Z_C} = +j5972\,\Omega$$

$$Z_{\text{EQ}} = 1000 + j5972\,\Omega = 6055\angle 80.5°\ \Omega$$

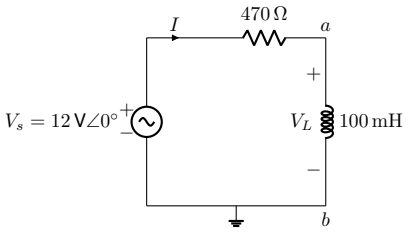
$$I = \frac{V_s}{Z_{\text{EQ}}} = \frac{20\angle 30°}{6055\angle 80.5°} = 3.303\angle -50.5°\ \text{mA}$$

$$i(t) = 3.303\cos(2\pi \cdot 1000t - 50.5°)\ \text{mA}$$

$\phi = 30° - (-50.5°) = +80.5°$: now **lagging**, so the pair is net *inductive*, and pf $= \cos 80.5° = 0.165$.

Read that parallel result again: $|Z_L \parallel Z_C| = 5972\,\Omega$ exceeds *both* branches (1257 and $1592\,\Omega$) — the two reactances have opposite signs and nearly cancel in the denominator, so carry the extra digit.

Your Turn



The same topology, now with an **inductor** instead of a capacitor: $V_s = 12\,\text{V}\angle 0°$ at $f = 500$ Hz.

1. Compute ω and Z_L
2. Compute Z_{EQ} in rectangular form, then convert to phasor form
3. Use the voltage divider to find V_L, then write $v_L(t)$
4. Does V_L lead or lag V_s? Does that match what you expect for an inductor?

Wrap-Up

Key ideas:

- A sinusoid at a known frequency *is* its phasor $A\angle\phi$ — amplitude and phase in one object
- **Add in rectangular, multiply in polar**, and always check the quadrant
- $Z_R = R, \quad Z_C = 1/j\omega C, \quad Z_L = j\omega L$, and impedance depends on **frequency**
- Capacitors block DC and pass highs; inductors do the reverse
- **Every Block 1 tool applies unchanged** — but a parallel pair can exceed both branches

Next lesson:

- **Lesson 17 — Objective 2.4: Analog Filters.** Building real filters out of R, L, and C
- **Quiz 4** (Objectives 2.1–2.2) is due at **Lesson 18** (28/29 Sep)
- The **Midterm** (Objectives 1.1–1.7, 2.1–2.3) is due at **Lesson 19** (30 Sep/1 Oct) — **today is the last new material on it**
- **Quiz 5** (Objectives 2.3–2.4) is due at **Lesson 21** (6/7 Oct)