

ECE 215/S – Principles of Electronic Cyber Warfare

Block 2: Signal Processing
Lesson 17: Analog Filters

Department of Electrical and Computer Engineering
United States Air Force Academy

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Objective 2.4

I can determine the filter type, gain, and cutoff frequency of simple series circuits containing a resistor, capacitor, and/or inductor.

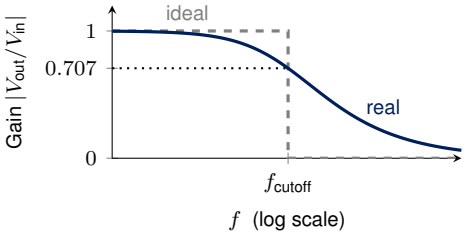
- Lesson 15 gave you **ideal** filters. Today: filters you can actually build
- Everything rests on Objective 2.3 — impedance changes with frequency
- Assessed on **Quiz 5** along with Objective 2.3 — *not* on the midterm

Where We Are Going

By the end of this lesson, you will be able to:

- Explain how a real filter differs from an ideal one
- Define the **cutoff frequency** at the -3 dB point
- Identify the **filter type** of an R-C, C-R, R-L, or L-R circuit *by inspection*
- Compute the **cutoff frequency** and the **gain** at any frequency

Ideal vs Reality



- An ideal filter transitions **instantaneously** from pass to stop — the dashed wall
- A real one **rolls off** gradually — there is no wall, only a slope
- Frequencies near the cutoff are **partially** attenuated
- Making the transition sharper costs more components, more size, and more money

Ideal filters are a design target. Rolloff is what you actually get.

The Cutoff Frequency

Since there is no wall, we need a convention for where "cutoff" is.

Cutoff is where output *power* falls to half its passband value.

- Half power means the **voltage** falls to $1/\sqrt{2} = 0.707$ of its passband value:

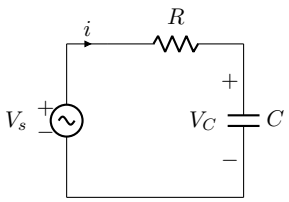
$$P_{\text{out}} = \frac{P_{\text{in}}}{2} \propto \left(\frac{V_{\text{in}}}{\sqrt{2}}\right)^2 = (0.707\, V_{\text{in}})^2$$

- As a **power** ratio, one half is $10\log_{10}(0.5) = -3$ dB — hence the -3 **dB point**. Power goes as V^2, so a **voltage** ratio converts with the equation sheet's gain row and lands on the same number:

$$G_{\text{dB}} = 20\log_{10}\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| \quad \Longrightarrow \quad 20\log_{10}(0.707) = -3 \text{ dB}$$

- "Cutoff frequency," "corner frequency," "half-power point," and "-3 dB point" all mean the same thing

The R-C Circuit: What Does It Do?



Take the output across the **capacitor**:

$$\frac{V_C}{V_s} = \frac{Z_C}{Z_{\text{EQ}}} = \frac{1}{j2\pi fRC + 1}$$

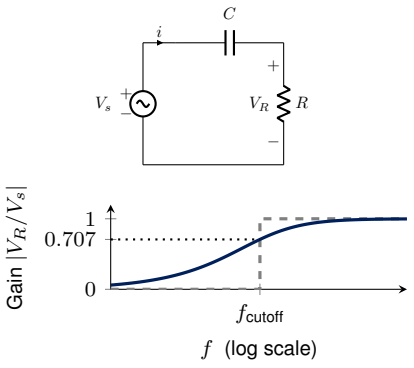
Check the two limits:

$$\lim_{f \to 0} \frac{V_C}{V_s} = \frac{1}{0+1} = 1$$

$$\lim_{f \to \infty} \frac{V_C}{V_s} = \frac{1}{j\infty} = 0$$

Passes low, blocks high — this is a **low-pass filter**.

The C-R Circuit



Swap the elements — now take the output across the **resistor**:

$$\frac{V_R}{V_s} = \frac{R}{R + Z_C} = \frac{1}{1 - j\dfrac{1}{2\pi f RC}}$$

- At **DC** the capacitor is an **open**: nothing reaches R, so gain $\to 0$
- At **high** f it is a **short**: all of V_s lands on R, so gain $\to 1$

Blocks low, passes high — a **high-pass filter**.

Same two parts. The filter type depends on where you take the output.

Cutoff Frequency for R-C and C-R

Both circuits share the same cutoff. We will write f_c for short.

$$f_{\text{cutoff}} = \frac{1}{2\pi RC}$$

- At f_c the two elements contribute equally — $|Z_C| = R$ — and the gain is 0.707
- There the phase is $45°$: the **low-pass** output **lags**, the **high-pass** output **leads** — the phase you will measure at Lesson 19
- Larger R *or* larger C both push f_c **down**
- **Example:** $R = 60\,\Omega$, $C = 1\,\mu$F gives $f_c = 1/[2\pi(60)(1 \times 10^{-6})] = 2.65$ kHz

Gain at any frequency

Low-pass (output across C):

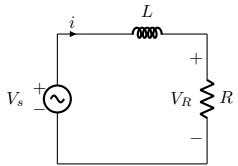
$$\left|\frac{V_C}{V_s}\right| = \frac{1}{\sqrt{1 + (f/f_c)^2}}$$

High-pass (output across R):

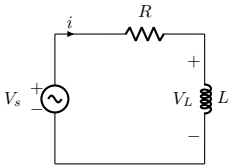
$$\left|\frac{V_R}{V_s}\right| = \frac{f/f_c}{\sqrt{1 + (f/f_c)^2}}$$

R-L and L-R Circuits

Inductors behave oppositely to capacitors, so the roles swap:



L-R: output across R
low-pass



R-L: output across L
high-pass

Output across	Type
R-C: capacitor	Low-pass
C-R: resistor	High-pass
R-L: inductor	High-pass
L-R: resistor	Low-pass

For the inductor circuits, $|Z_L| = R$ at cutoff — the same equal-contribution argument — which gives $f_{\text{cutoff}} = \dfrac{R}{2\pi L}$.

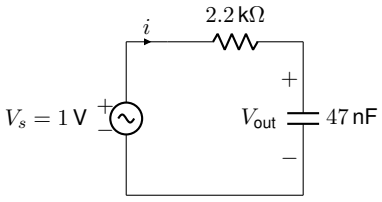
Method: Analyzing an R-C or R-L Filter

1. Name the **reactive element** (C or L) and the element the output is taken across
2. **Ask what that element does at DC** — capacitor $\rightarrow$ open, inductor $\rightarrow$ short — and whether the output **survives**: survives $\Rightarrow$ **low-pass (LPF)**, killed $\Rightarrow$ **high-pass (HPF)**
3. Compute the cutoff: $f_c = 1/(2\pi RC)$ for R-C and C-R; $f_c = R/(2\pi L)$ for R-L and L-R
4. Form the ratio f/f_c and evaluate the **gain magnitude**: $1/\sqrt{1+(f/f_c)^2}$ for a low-pass, $(f/f_c)/\sqrt{1+(f/f_c)^2}$ for a high-pass
5. **Check** the gain against the type — it approaches 1 in the passband, 0 in the stopband, and equals 0.707 at f_c

One resistor and one reactive element can only make an LPF or an HPF. Band-pass (BPF) and band-stop (BSF) need more parts.

Check DC. Whatever survives there names the filter.

Worked Example



Find:

1. The filter type
2. The cutoff frequency
3. The gain magnitude at $f = f_c$
4. The gain magnitude at $f = 10\,f_c$

Worked Example — Solution

1. Type
At DC the capacitor is an open, so all of V_s appears across it — the output *survives*. **Low-pass.**

2. Cutoff

$$f_c = \frac{1}{2\pi(2200)(47 \times 10^{-9})} = 1.54 \text{ kHz}$$

3. Gain at f_c
$f/f_c = 1$, so $\left|\frac{V_{\text{out}}}{V_s}\right| = \frac{1}{\sqrt{1+1^2}} = 0.707$ — that is -3 dB, which is what cutoff means.

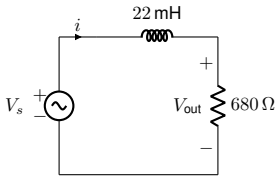
4. Gain at $10f_c$

$$\frac{f}{f_c} = 10 \implies \left|\frac{V_{\text{out}}}{V_s}\right| = \frac{1}{\sqrt{1+10^2}} = 0.0995$$

In decibels, $20\log_{10}(0.0995) = -20$ dB.

One **decade** — a factor of ten in frequency — costs a factor of ten in gain, which is 20 dB. That slope is what a *single* reactive element buys you, and it is called a **first-order** rolloff.

Your Turn



An L-R circuit, output across the **resistor**.

1. What is the filter type? Justify it using the DC test
2. What is the cutoff frequency?
3. What is the gain magnitude at $f = 1$ kHz?
4. You need the cutoff to be *half* its current value. What would you change, and to what?

Wrap-Up

Key ideas:

- Real filters **roll off** — there is no instantaneous transition
- **Cutoff** is the half-power (-3 dB) point, where the gain is 0.707 of its passband value
- $f_c = 1/(2\pi RC)$ for R-C and C-R; $f_c = R/(2\pi L)$ for R-L and L-R
- Gain at any f: $1/\sqrt{1+(f/f_c)^2}$ low-pass, $(f/f_c)/\sqrt{1+(f/f_c)^2}$ high-pass
- Identify the type by asking **what survives at DC**

Next up, and what is due:

- **Lesson 18 — Midterm Review**, then **Lesson 19 — the Analog Filter Lab**, where you build one of these and measure it
- The **midterm** is due at **Lesson 19**, covering Objectives **1.1–1.7 and 2.1–2.3** — today's Objective 2.4 is **not** on it. **Quiz 4** (Objectives 2.1–2.2) is due at **Lesson 18**, and **Quiz 5** (Objectives 2.3–2.4), which *does* cover today, at **Lesson 21** — 6 Oct / 7 Oct