

ECE 215/S – Principles of Electronic Cyber Warfare

Block 2: Signal Processing

Lesson 20: Analog-to-Digital Conversion

Department of Electrical and Computer Engineering
United States Air Force Academy

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Objective 2.5

I can calculate the ADC sampling rate, voltage resolution, and digital output based on a given input voltage and the ADC's operational parameters, and determine the number of bits a required resolution demands.

- Lesson 19 had you read filter voltages off the scope by hand, one frequency at a time. Today a computer does the reading — thousands of times a second
- This is the bridge between the analog world and everything a computer can do
- Assessed on **Quiz 6** along with Objective 2.6

Where We Are Going

By the end of this lesson, you will be able to:

- Describe the three stages of conversion: **sample**, **quantize**, **encode**
- Apply the **Nyquist criterion** to choose a sampling rate, and explain **aliasing**
- Compute **voltage resolution** from bit depth and input range
- Compute the **digital output value** for a given input voltage
- Bound the **quantization error**, and pick the bit depth an error spec demands

Analog vs Digital

Analog

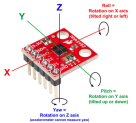
- Continuous in **time** and in **value**
- Infinitely many possible values
- What every physical sensor produces

Digital

- Discrete in **time** and in **value**
- A finite list of numbers
- What a computer can store and process

A **transducer** converts a physical quantity into a voltage — a thermometer, an accelerometer, a microphone.





The ADC Chain



- **Sample** — discretize in *time*. Take a snapshot every $T_s = 1/f_s$ seconds, where f_s is the **sampling rate**
- **Quantize** — discretize in *value*. Truncate each snapshot down to the level below it
- **Encode** — express that level as a binary number

Sampling loses time resolution. Quantizing loses amplitude resolution. Both are permanent.

Sampling Rate and Nyquist

$$f_s \geq 2f_{\text{high}}$$

- Sample at **at least twice** the highest frequency present — exactly $2f_{\text{high}}$ counts
- $2f_{\text{high}}$ is the **Nyquist rate** — the floor, not a target. f_{high} is the one you read off the Lesson 15 spectrum; a DC term sits at $f = 0$ and does **not** raise it
- **Examples:**
  - A wheel spinning at 35 Hz needs $f_s \geq 70$ Hz — TV samples at 60 Hz, so it aliases and the wheel rolls backward
  - Telephone voice to 3.4 kHz: $f_s \geq 6.8$ kHz (phones use 8 kHz)
  - Audio to 22 kHz: $f_s \geq 44$ kHz (CDs use 44.1 kHz)

Standards sit *above* the floor on purpose — a real anti-aliasing filter rolls off gradually (Objective 2.6).

Aliasing

Sample too slowly and a high frequency **masquerades** as a low one.

- The samples are consistent with more than one sinusoid — and the reconstruction picks the wrong one
- The false low frequency is called an **alias**
- **Aliasing is not recoverable.** Once the samples are taken, the information is gone; no processing brings it back
- You have seen it: the wheel from the last frame. A frame rate is a sampling rate, and when it sits below $2f_{\text{high}}$ for the spokes, the wheel rolls backward

The only cure is prevention — filter before you sample (Objective 2.6).

Quantization and Resolution

An ADC with b bits has a finite number of output levels:

$$\text{levels} = 2^b$$

$$\Delta V = \frac{V_{\max} - V_{\min}}{\text{levels}}$$

- ΔV is the **resolution** — the smallest voltage change the ADC can distinguish
- Every input is truncated down to the level below it. That lost fraction is the **quantization error** QE, and at worst it is a full step: $QE_{\max} = \Delta V$

Example: a 10-bit ADC over a 0–5 V range.

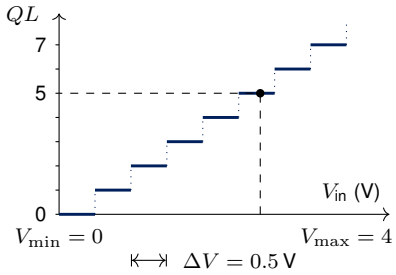
$$\text{levels} = 2^{10} = 1024 \qquad \Delta V = \frac{5\,\text{V} - 0\,\text{V}}{1024} = 0.00488\,\text{V} = 4.88\ \text{mV}$$

- **More bits means finer resolution.** Each additional bit halves ΔV

Encoding: The Digital Output

$V_{\text{in}} = 2.8\,\text{V} \Rightarrow EL = 5.6$

$QL = \lfloor 5.6 \rfloor = 5 = 101_2$



$b = 3$ bits, so $2^3 = 8$ levels, codes 0–7

$$EL = \frac{V_{\text{in}} - V_{\min}}{\Delta V}$$

$$QL = \lfloor EL \rfloor$$

- EL, the **expected level**, counts the resolution steps from the bottom of the range up to the input — rarely a whole number $(2.8/0.5 = 5.6)$
- QL, the **quantized level**, is the integer the ADC reports: **truncate**, never round. It is encoded in binary with b bits
- Inputs **outside** $[V_{\min}, V_{\max}]$ saturate at the extremes — that is **clipping** (Objective 2.6)

Method: Converting One Sample

1. **Check Nyquist:** $f_s \geq 2f_{\text{high}}$
2. **Get the resolution:** levels $= 2^b$, $\Delta V = (V_{\max} - V_{\min})/2^b$
3. **Locate the input:** $EL = (V_{\text{in}} - V_{\min})/\Delta V$ — keep ΔV unrounded, and **mind a negative** $V_{\min}$
4. **Truncate:** $QL = \lfloor EL \rfloor$, never round. Encode it in b bits
5. **Check:** $0 \leq QL \leq 2^b - 1$, and QE is positive and no larger than one step. A sample sitting exactly *at* $V_{\max}$ gives $EL = 2^b$ — **clamp it to the last code**, $2^b - 1$

The error itself

$$V_{\text{quant}} = QL\,\Delta V + V_{\min}$$

$$QE = V_{\text{in}} - V_{\text{quant}}, \quad QE_{\max} = \Delta V$$

Backward: bits from an error spec

$$2^b \geq \frac{V_{\max} - V_{\min}}{\Delta V_{\text{spec}}} = N, \quad b = \lceil \log_2 N \rceil$$

Round up — there is no partial bit, and rounding down leaves the error coarser than the spec

Bit Rate and Memory

Each sample costs b bits, and there are f_s of them per second:

bit rate $= b \times f_s$ **bits per second**

- Memory for a recording of duration t: bits $= b \times f_s \times t$
- Divide by 8 for bytes, and multiply by the number of **channels** for stereo or multi-sensor systems
- **Say which prefix you are using:** 1 kB $= 1000$ B (decimal), while the equation sheet's 1 kiB $= 2^{10}$ B $= 1024$ B (binary). Same bytes, different labels
- This is where the sampling-rate and bit-depth choices show up as a **cost**

Worked Example

A 12-bit ADC has an input range of 0 to 3.3 V. The signal has content up to 3 kHz.

Sampling rate

$$f_s \geq 2(3\text{ kHz}) = 6\text{ kHz}$$

Resolution

$$\text{levels} = 2^{12} = 4096$$

$$\Delta V = \frac{3.3\text{ V}}{4096} = 0.000806\text{ V} = 0.806\text{ mV}$$

Digital output for $V_{\text{in}} = 1.5$ V

$$EL = \frac{1.5\text{ V} - 0\text{ V}}{3.3\text{ V}/4096} = 1861.8$$

$$QL = \lfloor 1861.8 \rfloor = 1861$$

Divide by the *unrounded* ΔV.

Memory for 60 s at the standard $f_s = 8$ kHz (frame 6; above the 6 kHz floor)

$$12 \times 8000 \times 60 = 5.76\text{ Mbit}$$

$$= 720\text{ kB} = 703\text{ kiB}$$

Your Turn

An 8-bit ADC has an input range of -5 V to $+5$ V. The transducer signal is $V_M(t) = 2\cos(2\pi \cdot 3000\, t)$ V.

1. What is the minimum sampling rate?
2. What is the voltage resolution?
3. What quantized level QL is reported when $V_{\text{in}} = 2$ V?
4. If the maximum quantization error must be no more than 100 mV, what is the minimum b?
5. How much memory does a 10-second recording take at a chosen $f_s = 8$ kHz?

Wrap-Up

Key ideas:

- Conversion is three stages: **sample** (time), **quantize** (value), **encode** (binary)
- **Nyquist**: $f_s \geq 2f_{\text{high}}$ — at least twice. Too slow means **aliasing**, which is permanent
- levels $= 2^b$, $\Delta V = (V_{\max} - V_{\min})/2^b$. Backward: $2^b \geq (V_{\max} - V_{\min})/\Delta V_{\text{spec}}$, **round up**
- $QL = \lfloor EL \rfloor$ with $EL = (V_{\text{in}} - V_{\min})/\Delta V$ — **mind a negative** $V_{\min}$
- $V_{\text{quant}} = QL\,\Delta V + V_{\min}$, and $QE = V_{\text{in}} - V_{\text{quant}}$ is never worse than a step: $QE_{\max} = \Delta V$

Next lesson:

- **Lesson 21 — Objective 2.6: Signal Conditioning.** Getting the transducer's signal to actually fit the ADC
- **Quiz 5** (Objectives 2.3–2.4) is due at Lesson 21