

ECE 215/S – Principles of Electronic Cyber Warfare

Block 3: Communication Systems, Radar, and Electronic Warfare

Lesson 29: AM Bandwidth and Multiplexing

Department of Electrical and Computer Engineering
United States Air Force Academy

Fall 2026

Objective 3.1 (completing)

I can determine the modulation index, output signal, and output signal bandwidth of an Amplitude Modulation (AM) system and assess whether the system is under-modulated, over-modulated, or fully modulated.

- Lesson 28 gave you the three-term AM output and $\alpha = \max(\text{message})/B$
- Today: what each value of α **does to the waveform**, how to find **bandwidth**, and how many signals fit in a band
- Assessed on **Quiz 8** (Objectives 3.1–3.2), due at **Lesson 32**

Where We Are Going

By the end of this lesson, you will be able to:

- Identify **under-**, **fully**, and **over-modulated** signals from a waveform or from α
- Explain the **crossover point** and why over-modulation defeats an envelope detector
- Draw the output **spectrum** and compute the **bandwidth** for single-tone, multi-tone, and continuous messages
- Describe **frequency division multiplexing** and lay out a set of channels

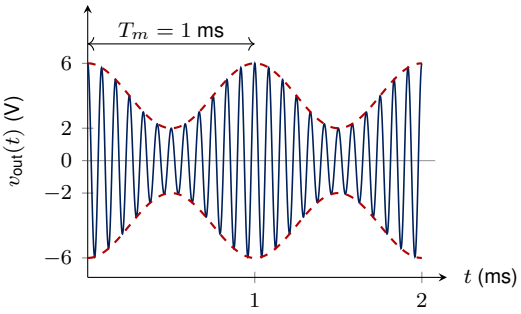
The Crossover Point

The AM envelope is $A_c\big[B + v_m(t)\big]$ — the biased message, scaled by the carrier amplitude. The question that decides everything:

Does the envelope ever cross zero?

- As long as the envelope stays **above zero**, its shape *is* the message, and a cheap detector can recover it
- The **crossover point** is where the envelope reaches zero. Past it the envelope folds through, the carrier flips phase — and the outline stops matching the message
- That threshold is exactly $\alpha = 1$: a single tone's envelope bottoms out at $A_c(B - A_m)$, which is zero when $A_m = B$

Under-Modulation ($\alpha < 1$)



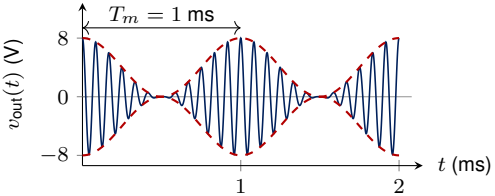
A 1 kHz tone on a 12 kHz carrier, with $A_c = 1$ V

Here $B = 4$ V and $A_m = 2$ V, so $\alpha = 2/4 = 0.5$.

- Envelope swings between $A_c(B - A_m) = 2$ V and $A_c(B + A_m) = 6$ V
- It **never reaches zero**, so an envelope detector recovers the message
- From the envelope,
$$\alpha = \frac{\max - \min}{\max + \min} = \frac{6 - 2}{6 + 2} = 0.5$$
- But a lot of power sits in the carrier doing nothing (Lesson 30 puts a number on it)

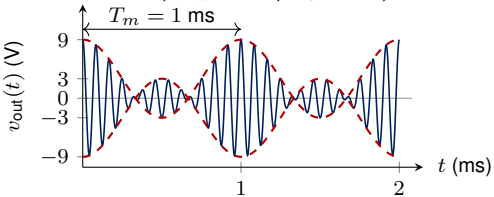
Full and Over-Modulation ($\alpha = 1$ and $\alpha > 1$)

Full: $B = A_m = 4\,\text{V}$, $A_c = 1\,\text{V}$, so $\alpha = 1$



- Envelope **just touches zero**: $A_c(B - A_m) = 0$
- The most efficient case an envelope detector can still handle (Lesson 30 puts a number on it)

Over: $B = 3\,\text{V}$, $A_m = 6\,\text{V}$, $A_c = 1\,\text{V}$, so $\alpha = 2$



- $A_c(B - A_m) = -3\,\text{V}$: the envelope **crosses zero** and folds, so the outline no longer traces the message
- An envelope detector gives *garbage*; only a **synchronous** one works (Lesson 30)

Over-modulation is not a little worse. It breaks the cheap receiver entirely.

AM Bandwidth

A single tone at f_m puts sidebands at $f_c \pm f_m$, so $BW = f_{high} - f_{low} = (f_c + f_m) - (f_c - f_m)$:

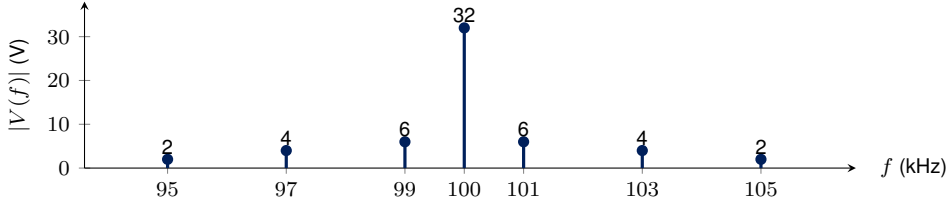
$$BW = 2f_m$$

- Bandwidth is set by the **message**, not the carrier. A 5 kHz message needs 10 kHz of spectrum whether the carrier is at 1 MHz or 10 GHz
- The carrier only decides *where* that 10 kHz block sits
- The bias rides at f_c, **not** at $f = 0$. Lesson 15's "a DC term makes $f_{low} = 0$" is about the message; in the AM output, $f_{low} = f_c - f_m$
- For a message with several tones, use the **highest** one: $BW = 2f_{m,\max}$

Multiple Message Frequencies

A real message is not one tone. Bias this one by $B = 8$ V and put it on a 4 V carrier at 100 kHz:

$$v_m = 3\cos(2\pi \cdot 1000t) + 2\cos(2\pi \cdot 3000t) + \cos(2\pi \cdot 5000t) \text{ V}$$

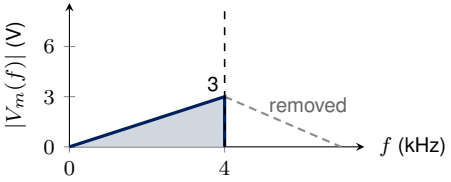


- **Every tone appears twice, mirrored about the carrier**, scaled by $A_c/2 = 2$
- A bias is present, so the **carrier** is too: $A_c B = (4)(8) = 32$ V at 100 kHz
- The cosines peak together at $t = 0$: $\max(\text{message}) = 3 + 2 + 1 = 6$ V, so $\alpha = 6/8 = 0.75$
- $BW = 105 - 95 = 10$ kHz $= 2 \times 5$ kHz — set by the **highest** tone

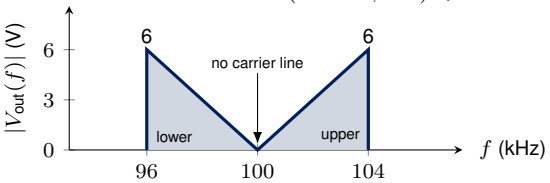
Non-Sinusoidal Messages

Voice, music, and data are not sinusoids — but any signal is a **sum of sinusoids** (Objective 2.1), so its spectrum is a **continuous curve**: a tone at every frequency under it.

Message, after an LPF with $f_{\text{cutoff}} = 4$ kHz



After the mixer: carrier $4\cos(2\pi \cdot 100{,}000t)$ V, **no bias**

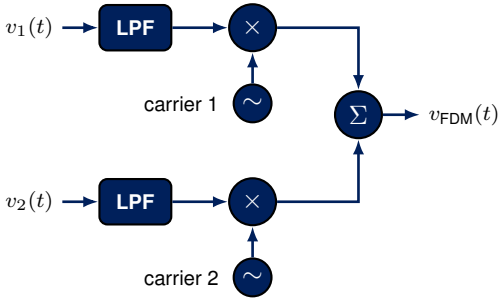


- Each component is a tone: it moves to $f_c \pm f$, scaled by $A_c/2 = 2$, so 3 V becomes 6 V
- **Upper sideband:** the message's shape, shifted up by f_c. **Lower:** its **mirror image**
- **No bias, no carrier line.** A bias B would add one stem of height $A_c B$ at 100 kHz
- $BW = 104 - 96 = 8$ kHz $= 2f_{m,\max}$, and the LPF sets $f_{m,\max} = f_{\text{cutoff}}$

Frequency Division Multiplexing

If each signal occupies a known block of spectrum, you can stack many of them side by side:

- Each channel gets its **own carrier**
- An **LPF** band-limits each message first, so each block is $BW = 2f_{\text{cutoff}}$ wide
- Space the carriers at least BW apart so the blocks do not overlap. Real systems add a **guard band**, because filters are not ideal (Objective 2.4)
- A **summer** adds the channels into one signal
- The receiver tunes a band-pass filter to the channel it wants and ignores the rest



An FDM transmitter: band-limit, shift, add. A channel with a bias has a Σ ahead of its mixer, as in Lesson 28.

This is why your radio has a dial. Every station is one FDM channel.

Method: Laying Out an FDM Band

To place channels in a band and draw the result:

1. **Highest message frequency:** $f_{m,\max}$ — the top tone, or the band-limiting LPF's f_{cutoff}
2. **Channel bandwidth:** $BW = 2f_{m,\max}$
3. **Carriers:** space them at least BW apart, plus any guard band
4. **Blocks:** each channel runs from $f_c - f_{m,\max}$ to $f_c + f_{m,\max}$ — the message's shape above f_c scaled by $A_c/2$, its mirror image below, and a carrier stem $A_c B$ only if that channel is biased
5. **Total spectrum:** $f_{high} - f_{low}$ across every channel
6. **Check:** no two blocks overlap, and each block is symmetric about its own carrier

Worked Example

Three voice channels are to be multiplexed. An LPF with $f_{\text{cutoff}} = 4$ kHz band-limits each one, and the first carrier is at 100 kHz.

Find:

1. The bandwidth of each channel
2. The minimum carrier spacing
3. Carrier frequencies for channels 2 and 3
4. The band each channel occupies, and the total spectrum

Work it before the next slide.

Assume no guard band for now — then think about what a real system would add.

Worked Example — Solution

1. $f_{m,\max} = f_{\text{cutoff}} = 4$ kHz

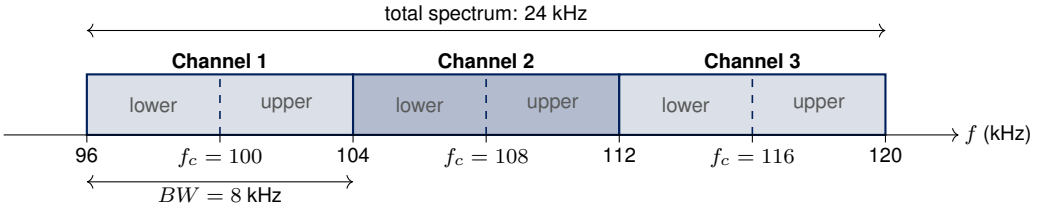
2. Channel bandwidth: $BW = 2(4) = 8$ kHz

3. Carriers 8 kHz apart, so the blocks just touch: $f_c = 100$, 108, and 116 kHz

4. Blocks run $f_c \pm 4$ kHz: 96–104, 104–112, and 112–120 kHz

5. Total: $120 - 96 = 24$ kHz $= 3 \times BW$

6. Check: the blocks meet at 104 and 112 kHz, but none overlap



A real system would space these further apart — real filters cannot cut off instantly.

Your Turn

An AM transmitter uses a 600 kHz carrier with $A_c = 10$ V. The message is $v_m = 3\cos(2\pi \cdot 2000t) + 1.5\cos(2\pi \cdot 6000t)$ V, with a bias of $B = 5$ V.

1. What is the modulation index? Is the signal under-, fully, or over-modulated?
2. List every frequency present at the output **and its amplitude**, and draw the spectrum
3. What is the bandwidth?
4. How many such channels fit between 530 kHz and 1700 kHz?
5. A second channel puts the earlier ramp message (0 V at 0 Hz rising to 3 V at 4 kHz) on a 4 V carrier at 620 kHz, with **no bias**. Add it to your spectrum

Wrap-Up

Key ideas:

- $\alpha < 1$ under-, $\alpha = 1$ fully, $\alpha > 1$ over-modulated. Past $\alpha = 1$ the envelope crosses zero, which defeats an envelope detector
- Every message component appears **twice**, mirrored about f_c and scaled by $A_c/2$. A bias adds the carrier line $A_c B$; no bias, no line
- $BW = 2f_{m,\max}$ — set by the **message**, not the carrier
- **FDM** stacks channels on separate carriers, spaced by at least BW plus a guard band

Due at this lesson:

- **Project 2** — exact deadline on Gradescope

Next lesson:

- **Objective 3.1 is now complete. Quiz 8** (Objectives 3.1–3.2) is due at **Lesson 32 (6/9 Nov)**
- **Lesson 30 — Objective 3.2: Demodulation.** Getting the message back out — and the last new material before Quiz 8