

ECE 215/S – Principles of Electronic Cyber Warfare

Block 3: Communication Systems, Radar, and Electronic Warfare

Lesson 33: The Friis Equation

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Objective 3.5

I can calculate the maximum communication distance between a transmitter and receiver using the Friis and Line-of-Sight equations.

- Lesson 32 answered “can they see each other?” Today: “is there enough power when it arrives?”
- Both must be true for a link to close
- Assessed on **Quiz 9** along with Objectives 3.3 and 3.4, and again on **Quiz 10**

Where We Are Going

By the end of this lesson, you will be able to:

- Write the **Friis equation** and explain where each term comes from
- Predict how received power changes when power, gain, frequency, or range changes
- Solve Friis for the **maximum range** at which a receiver still gets the power it needs
- Work a complete **radio communication problem**, combining Friis with LOS

The Link Budget Problem

Two radios, some distance apart. Will they talk?

- The transmitter puts out P_T watts
- The receiver needs at least $P_{R,\min}$ watts to make sense of the signal — its **sensitivity**
  - A specification table may call it the receiver's "received power" or "minimum power." It is the **threshold**, not what actually arrives
- Everything in between either **adds** gain or **takes away** loss
- Add it all up. If what arrives **meets or exceeds** $P_{R,\min}$, the link closes

A link budget is bookkeeping on power, exactly like Block 1 — just over the air.

The Friis Equation

$$P_R = P_T \, G_T \, G_R \, \frac{\lambda^2}{(4\pi R)^2}$$

P_R	power collected by the receiving antenna (W)
P_T	power sent by the transmitting antenna (W)
G_T	gain of the transmitting antenna — a **linear ratio**, no units
G_R	gain of the receiving antenna — a **linear ratio**, no units
λ	wavelength of the signal, $\lambda = c/f$ (m)
R	distance between the two antennas (m) — a **range** now, not a resistance

The equation sheet writes the last factor as $(\lambda/4\pi R)^2$ — the same thing.

Where Each Term Comes From

Build it in three steps — the same bookkeeping as a power chain:

1. **Transmit.** P_T watts, focused by gain G_T, spread over a sphere of radius R:

$$S = \frac{P_T G_T}{4\pi R^2} \quad \text{W/m}^2$$

2. **Collect.** The receiving antenna captures an area — its **effective aperture**:

$$A_e = \frac{\lambda^2}{4\pi} G_R$$

3. **Multiply.** Power density times capture area:

$$P_R = S \cdot A_e = P_T G_T G_R \frac{\lambda^2}{(4\pi R)^2}$$

The $1/R^2$ is nothing but the surface area of an expanding sphere.

Implications

Read the equation as a set of levers:

- **Double** $P_T \rightarrow$ double P_R
- **Double** G_T **or** $G_R \rightarrow$ double P_R
- **Double** λ (that is, *halve* the frequency) $\rightarrow$ **four times** P_R
  - Lower frequencies travel **farther** for the same power
- **Double** $R \rightarrow$ **quarter** the P_R

Range and wavelength enter squared. Power and gain do not.

But recall Lesson 31: dish gain goes as $1/\lambda^2$. Raising frequency costs you here and pays you back in antenna gain — a real design tension.

Decibels: A Refresher

Link budgets span enormous ranges, so specification sheets often quote gains and powers in **dB**. In this course every gain you put *into an equation* is a **linear ratio** — convert first.

$$X_{\text{dB}} = 10\log_{10}(X) \qquad X = 10^{X_{\text{dB}}/10}$$

- **dBW** — power referenced to 1 W
- **dBm** — power referenced to 1 mW
- 0 dBW $= 30$ dBm

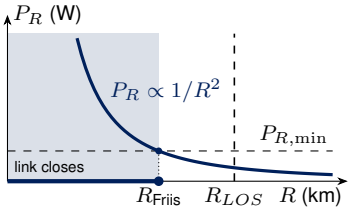
Power ratios worth memorizing

Ratio	dB
$\times 2$	$+3$
$\times 4$	$+6$
$\times 10$	$+10$
$\times 100$	$+20$
$\div 2$	-3

Power ratios use $10\log_{10}$. The equation sheet's $G_{dB} = 20\log_{10}(G_{abs})$ row is the **voltage-gain** form from Lesson 17 — do not use it on a power or on an antenna gain.

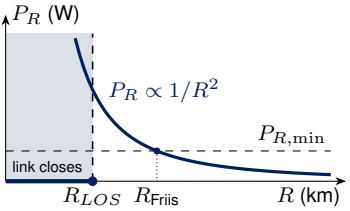
Combining Friis with LOS

A link has **two** range limits: the horizon, R_{LOS} (Lesson 32), and the power budget, R_{Friis} — the range at which P_R has fallen to $P_{R,\min}$. The maximum range is the **smaller** one.



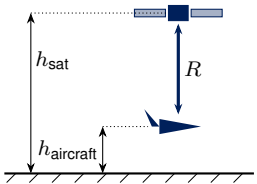
Power binds

$R_{\text{Friis}} < R_{LOS}$



LOS binds

$R_{LOS} < R_{\text{Friis}}$



Overhead: no horizon

$R = h_{\text{sat}} - h_{\text{aircraft}}$

Either limit can bind. Compute both — the smaller is your answer.

Method: The Radio Communication Problem

1. **Read the problem.** "Does the link close at this range?" or "what is the maximum range?" List the givens — each gain is the one *toward the other antenna* — and find $\lambda = c/f$
2. **LOS limit.** $R_{LOS,mi} = \sqrt{2\,h_{ft,1}} + \sqrt{2\,h_{ft,2}}$ — Lesson 32's row, straight off the equation sheet. Feet in, **miles out**; then $\times\, 1.61$ km/mi
   - Satellite directly overhead: no horizon. Use $R = h_{\text{sat}} - h_{\text{aircraft}}$, with feet $\times\, 0.305$ m/ft
3. **Power limit.** $R_{\text{Friis}} = \dfrac{\lambda}{4\pi}\sqrt{\dfrac{P_T\,G_T\,G_R}{P_{R,\min}}}$ — the sheet's Friis row solved for R; meters out
   - Asked about one given range instead? Compute P_R there and compare it with $P_{R,\min}$
4. **Maximum range** $=$ the **smaller** of the two limits, in the same unit
5. **Check:** put it back into Friis. $P_R \geq P_{R,\min}$ — equal if power binds, larger if LOS binds

Worked Example

A microwave relay link between two dishes, each on top of a **350-foot tower**. The towers are $R = 80$ km apart.

P_T	120 W	$P_{R,\min}$	200 nW
G_T	1201.2 (linear)	G_R	281.49 (linear)
f	3.7 GHz	$h_1 = h_2$	350 ft

1. Find λ
2. Do the two dishes have line of sight at 80 km?
3. Find the received power at 80 km. Does the link close?
4. What is the **maximum range** of this link, and which limit binds?

Worked Example — Solution

Step 1 — wavelength

$$\lambda = \frac{3 \times 10^8}{3.7 \times 10^9} = 0.0811 \text{ m}$$

Step 2 — LOS limit

$$R_{LOS,mi} = \sqrt{2(350)} + \sqrt{2(350)} = 52.92 \text{ mi}$$

52.92 mi $\times$ 1.61 km/mi $= 85.2$ km. The dishes are 80 km apart, so there **is** line of sight.

Step 3 — power at 80 km

$$P_R = \frac{(120)(1201.2)(281.49)(0.0811)^2}{(4\pi \cdot 80{,}000)^2} = 264 \text{ nW}$$

264 nW $\geq$ 200 nW, so the link **closes**.

Step 3 — power limit (the sheet's solved form)

$$R_{\text{Friis}} = \frac{0.0811}{4\pi}\sqrt{\frac{(120)(1201.2)(281.49)}{200 \times 10^{-9}}}$$

$$R_{\text{Friis}} = 91{,}900 \text{ m} = 91.9 \text{ km}$$

Step 4 — maximum range
The smaller of 85.2 km and 91.9 km:

$$R_{\max} = 85.2 \text{ km} \quad \textbf{— LOS binds}$$

The last 6.7 km of power budget lies beyond the horizon.

Step 5 — check
At 85.2 km Friis gives $P_R = 233$ nW ≥ 200 nW. ✓

Your Turn

A UAV at **10,000 feet** relays to a ground station whose antenna is **50 feet** up. The UAV transmits $P_T = 5$ W at $f = 2$ GHz with $G_T = 4$; the ground antenna has $G_R = 100$ and sensitivity $P_{R,\min} = 1$ nW. All gains are linear.

1. What is the maximum LOS distance, in km?
2. What is the maximum range from the power budget alone?
3. What is the actual maximum communication range, and which limit binds?
4. What single change would you make to extend it?
5. The UAV also uplinks to a satellite **directly overhead** at an altitude of **400 km**, with the same transmitter and gain. The satellite has $G_R = 100$ and $P_{R,\min} = 1.5$ pW. What range goes into Friis, and does the uplink close?

Wrap-Up

Key ideas:

- $P_R = P_T G_T G_R \lambda^2/(4\pi R)^2$ — power spread over a sphere, collected by an aperture
- Gains are **linear**. Double the range, quarter the power; halve the frequency, quadruple it
- **Power limit:** Friis solved for R at $P_R = P_{R,\min}$. **LOS limit:** $R_{LOS,mi}$ — feet in, miles out, $\times 1.61$ for km
- The maximum range is the **smaller** limit, and **either can bind** — compute both

Next lesson:

- **Lesson 34 — Objective 3.6: The Radar Range Equation.** What changes when the signal has to make the trip *twice*
- **Quiz 9** (Objectives 3.3–3.5) is due at **Lesson 35 (17/18 Nov)** — today is the **last new material on it**; Lesson 34's Objective 3.6 is on Quiz 10