

ECE 215/S – Principles of Electronic Cyber Warfare

Block 3: Communication Systems, Radar, and Electronic Warfare

Lesson 34: The Radar Range Equation

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Objective 3.6

I can calculate the maximum detection range from a radar antenna to a target using the Radar Range equation.

- Friis (Lesson 33) handled a **one-way** link. Radar has to make the trip **twice**
- That single change dominates everything about how radar behaves
- Assessed on **Quiz 10** along with Objectives 3.5 and 3.7–3.9

Where We Are Going

By the end of this lesson, you will be able to:

- Explain what **radar cross section** σ measures, and read it off an RCS plot
- **Derive** the radar range equation in three steps from Friis
- Solve for **maximum detection range** and explain the fourth-root behavior
- Contrast a **two-way** radar link with a **one-way** intercept by a radar warning receiver, and decide who detects first

From Comms to Radar

Communications (one way)

- Transmitter $\rightarrow$ receiver
- The receiver *wants* the signal and has its own antenna and amplifier
- Spreading loss applies **once**: $1/R^2$

Radar (two way)

- Transmitter $\rightarrow$ target $\rightarrow$ same antenna
- The target does not cooperate — it just *scatters* whatever hits it
- Spreading loss applies **twice**: $1/R^4$

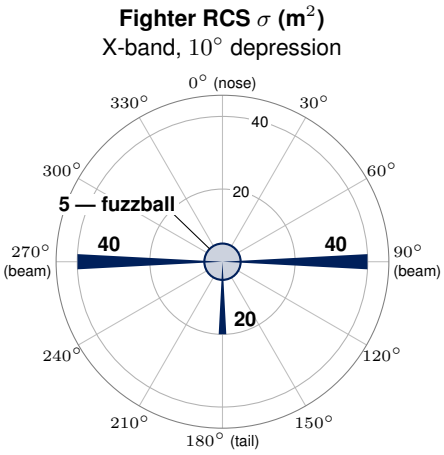
Everything hard about radar follows from that second trip.

Radar Cross Section

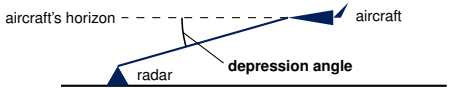
A radar's echo depends on the radar, on the range — and on one property of the target itself.

- **Radar cross section** σ quantifies how much power a target scatters back — how "big" it looks to a radar. The equation sheet writes it **RCS**
- Units of **area** (m^2), but it is *not* physical size. It is defined so that "power captured and re-radiated isotropically" comes out right
- σ varies enormously with **aspect** — the direction the radar views the target from — and with **shape** and **surface coatings**
- A shaped, coated aircraft can present a far smaller σ than its physical size suggests — that is the entire premise of low observability

Reading an RCS Plot



- **Angle** $=$ the **aspect**: where the radar is, as seen from the aircraft (0° nose, 180° tail). **Radius** $= \sigma$ in m^2, on a linear scale
- **Spikes:** narrow aspects where an edge or flat surface throws energy straight back. Radar on the beam (90°): $\sigma = 40\ \text{m}^2$
- **Fuzzball:** the low level at *every other* aspect. Radar dead ahead (0°): no spike there, so $\sigma = 5\ \text{m}^2$
- **No aspect named in the problem? Use the fuzzball**
- A plot is drawn for one **frequency band** and one **depression angle** — how far below the aircraft's horizon the radar sits, looking up:



Building the Equation

Three steps — Friis, run out and back. The radar's one antenna has gain G both ways.

1. **Out.** Power *density* reaching the target: $S = \dfrac{P_T\, G}{4\pi R^2}$ (W/m^2) — one sphere of spreading
2. **Scatter.** The target sends back $P_{\text{reflected}} = \sigma \cdot S$ (W) — that is the job σ does
3. **Back.** Another sphere, then the antenna's aperture $A_e = \dfrac{G\lambda^2}{4\pi}$ collects: $P_R = \dfrac{P_{\text{reflected}}}{4\pi R^2}\, A_e$

$$P_R = \frac{P_T\, G^2\, \sigma\, \lambda^2}{(4\pi)^3\, R^4}$$

- G^2: the *same* antenna transmits and receives. The sheet prints G_T^2, and (RCS) for σ
- $(4\pi)^3$: two spheres and one aperture. R^4 is the headline term

Maximum Detection Range

A radar detects when the echo meets or exceeds its **sensitivity** $P_{R,\min}$ — its *minimum detectable signal*. Set $P_R = P_{R,\min}$ and solve for R:

$$R_{\max} = \sqrt[4]{\frac{P_T\, G^2\, \sigma\, \lambda^2}{(4\pi)^3\, P_{R,\min}}}$$

- **Every term sits under a fourth root**
- To **double** detection range you must raise P_T by a factor of **16**
- Cutting σ by 16 only **halves** the range
- $P_{R,\min}$ is the same idea as in Lesson 33. Lesson 36 shows where the number comes from

Why the Fourth Root Matters

The fourth root is brutal on the radar designer and merciful to the target.

Term	To double R	Practical?
Transmit power P_T	$\times 16$	Expensive; thermal and prime-power limits
Antenna gain G	$\times 4$ (it is squared)	Huge apertures; physically bounded
Sensitivity $P_{R,\min}$	$\div 16$	Bounded by the noise floor (Lesson 36)
Target RCS σ	$\times 16$	Not the radar's to choose — it is the **target's** lever

- The radar pays $\times 16$ for every range doubling, and there is no cheap term left to push
- The target pulls the same lever the other way: cutting σ by 16 **halves** the range — and shaping and coatings are one-time engineering, not a power bill

Both sides pay dearly. Stealth wins because its lever is cheaper to keep pulling.

The Radar Warning Receiver

A radar warning receiver (RWR) does *not* play the two-way game. It just listens.

Radar detecting the aircraft	two-way	$P_R \propto 1/R^4$	the radar range equation
RWR detecting the radar	one-way	$P_R \propto 1/R^2$	plain Friis

- **RWR range:** Friis solved for R. The radar is the transmitter, so G_T is the radar's G; G_R and $P_{R,\min}$ are the RWR's own
- The aircraft *often* hears the radar before the radar sees it — but not always. It depends on the two sensitivities and on σ
- Listening is free, because the radar has to transmit to do its job — the foundation of electronic support (Lesson 37)

$1/R^2$ one way, $1/R^4$ two ways. Compute both ranges to see who detects first.

Method: The Radar Detection Problem

1. **Read the problem.** Radar: P_T, G, f, $P_{R,\min}$. Target: σ — the fuzzball unless an aspect is named. RWR: G_R and its own $P_{R,\min}$. Find $\lambda = c/f$
2. **Radar range (two-way).** $R_{\text{radar}} = \sqrt[4]{\dfrac{P_T\, G^2\, \sigma\, \lambda^2}{(4\pi)^3\, P_{R,\min}}}$ — the sheet's radar row solved for R
   - Asked about one given range instead? Compute P_R there and compare it with $P_{R,\min}$
3. **RWR range (one-way).** $R_{\text{RWR}} = \dfrac{\lambda}{4\pi}\sqrt{\dfrac{P_T\, G\, G_R}{P_{R,\min}}}$ — Friis, with the RWR's G_R and $P_{R,\min}$
4. **LOS cap.** $R_{LOS,mi} = \sqrt{2\, h_{ft,1}} + \sqrt{2\, h_{ft,2}}$, then $\times\, 1.61$ km/mi — nobody detects beyond it
5. **Compare.** The **longer** range detects first
6. **Check:** each range, put back into its own equation, returns that receiver's $P_{R,\min}$

Worked Example

A radar on a **100-foot** hill has $P_T = 1000$ W, $G = 150$, $f = 300$ MHz, and sensitivity $P_{R,\min} = 10$ fW.
(1 fW $= 10^{-15}$ W. Femto and atto are in the equation sheet's prefix table.)

A bomber with $\sigma = 100$ m^2 flies at **30,000 feet**, $R = 86$ km from the radar. Its RWR has $G_R = 3$ and sensitivity $P_{R,\min} = 215$ nW. All gains are linear.

1. Find the wavelength
2. Find the echo power at 86 km. Is the bomber detected?
3. Find the radar's maximum detection range
4. Find the range at which the RWR detects the radar
5. Does line of sight limit either range? Who detects first?

Worked Example — Solution

Step 1 — wavelength

$$\lambda = \frac{3 \times 10^8}{300 \times 10^6} = 1 \text{ m}$$

Step 2 — echo power at 86 km

$$P_R = \frac{(1000)(150)^2(100)(1)^2}{(4\pi)^3(86{,}000)^4} = 20.73 \text{ fW}$$

20.73 fW ≥ 10 fW, so the bomber **is detected**.

Step 2 — radar range

$$R_{\text{radar}} = \sqrt[4]{\frac{(1000)(150)^2(100)(1)^2}{(4\pi)^3(10 \times 10^{-15})}} = 103.2 \text{ km}$$

Step 3 — RWR range (Friis, one-way)

$$R_{\text{RWR}} = \frac{1}{4\pi}\sqrt{\frac{(1000)(150)(3)}{215 \times 10^{-9}}} = 115.1 \text{ km}$$

Step 4 — LOS cap

$$R_{LOS,mi} = \sqrt{2(100)} + \sqrt{2(30{,}000)} = 259.1 \text{ mi}$$

$\times$ 1.61 km/mi $= 417.1$ km. Both ranges are well inside it, so LOS limits neither.

Step 5 — compare. 115.1 km $>$ 103.2 km: the **bomber hears the radar first**, about 12 km before the radar sees it.

Step 6 — check. Echo at 103.2 km: 10.0 fW ✓
RWR at 115.1 km: 215 nW ✓

Your Turn

An X-band ($f = 10$ GHz) fire-control radar has $P_T = 10$ kW, $G = 2000$, and sensitivity $P_{R,\min} = 1$ fW. The fighter from the *Reading an RCS Plot* slide flies **straight at it**, nose-on.

1. What σ do you read off the plot, and what is the maximum detection range?
2. Shaping and coatings cut that RCS to $\sigma = 0.05$ m^2. What is the new range? By what factors did RCS and range drop?
3. The fighter's RWR has $G_R = 2$ and sensitivity $P_{R,\min} = 60$ nW. At what range does it detect the radar?
4. Who detects first — before the RCS reduction, and after it?
5. Why does the cut matter operationally, even though the range reduction looks modest?

Wrap-Up

Key ideas:

- **RCS** σ measures scattered power in m^2 — not physical size. On an RCS plot, use the **fuzzball** unless an aspect is named
- $P_R = P_T G^2 \sigma \lambda^2 / \left[(4\pi)^3 R^4\right]$ — two-way spreading gives R^4
- $R_{\max}$ puts **every term under a fourth root**: doubling range costs $\times 16$ in power
- An **RWR** hears the radar one-way (Friis). Compute both ranges to see who detects first

Next lesson:

- **Lesson 35 — Objective 3.7: Pulse-Doppler Radar.** Not just *whether* there is a target, but how far and how fast
- **Quiz 9** (Objectives 3.3–3.5) is due at **Lesson 35 (17/18 Nov)**; nothing from today is on it. Objective 3.6 is on **Quiz 10** (Objectives 3.5–3.9), due at **Lesson 38 (1/2 Dec)**