

ECE 215/S – Principles of Electronic Cyber Warfare

Block 3: Communication Systems, Radar, and Electronic Warfare

Lesson 35: Pulse-Doppler Radar

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Objective 3.7

I can calculate the range resolution, unambiguous range, and target velocity for pulse Doppler radar.

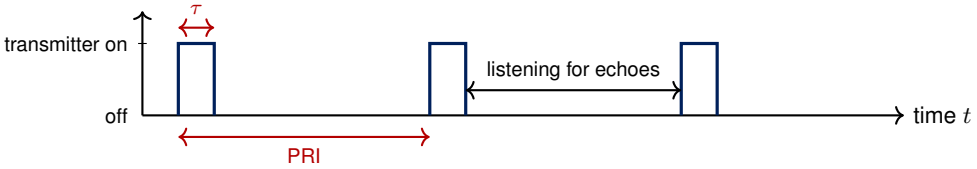
- Lesson 34 told you *whether* a target is detectable. Today: **how far** and **how fast**
- Assessed on **Quiz 10** along with Objectives 3.5, 3.6, 3.8, and 3.9

Where We Are Going

By the end of this lesson, you will be able to:

- Define **pulse width** τ, **PRF**, and **PRI**, and relate them
- Compute **range resolution** ΔR and explain what sets it
- Compute **maximum unambiguous range**, and where a target beyond it is reported
- Use the **Doppler equation** both ways: return frequency from speed, and speed and direction from a frequency shift

Pulsed Radar: The Three Quantities



- **Pulse width** τ — how long the transmitter is on. Sets *resolution*
- **PRI** (pulse repetition interval) — time from the start of one pulse to the start of the next. Sets *unambiguous range*
- **PRF** (pulse repetition frequency) — pulses per second

$$PRI = \frac{1}{PRF}$$

Range to a Target

The radar measures the **round-trip time** Δt of the echo:

$$R = \frac{c\,\Delta t}{2}$$

- The factor of 2 is there because the pulse travels **out and back**
- Light covers about 300 m per microsecond, so a target 15 km away returns its echo 100 μs later
- This row is not on the equation sheet, but its two children are. Put in τ for the time and you get **range resolution**; put in PRI and you get **unambiguous range**

Range Resolution

Two targets are **resolvable** only if their echoes do not overlap in time.

$$\Delta R = \frac{c\,\tau}{2}$$

Ideal case
Targets separated by more than ΔR produce two distinct returns. The radar reports two targets.

When it fails
Closer than ΔR, the echoes overlap and merge. The radar reports **one** target.

- A $1\ \mu$s pulse gives $\Delta R = 150$ m — two aircraft within 150 m appear as one
- A simple pulse radar's receiver bandwidth is matched to its pulse: $BW \approx 1/\tau$
- **Shorter pulse $\rightarrow$ finer resolution.** The price: less energy in each pulse, so a weaker echo and less detection range. Lesson 36 shows what the echo has to beat

Maximum Unambiguous Range

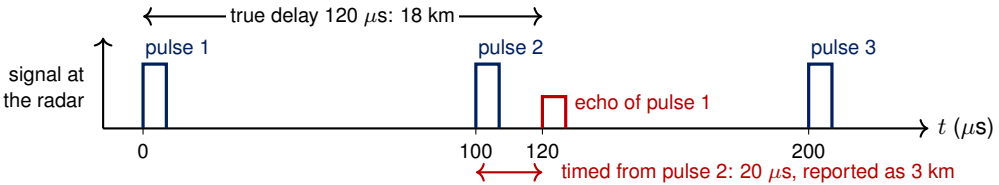
The radar must hear the echo **before** it sends the next pulse.

$$R_{\text{unamb}} = \frac{c(PRI)}{2}$$

- This is the equation sheet's **Unambig Range** row, with the same symbol
- A radar with $PRF = 1$ kHz has $PRI = 1$ ms, so $R_{\text{unamb}} = 150$ km
- **Lower PRF $\rightarrow$ longer PRI $\rightarrow$ greater unambiguous range.** The price: fewer pulses reach the target each second, so tracking and Doppler measurement suffer
- Pulse width sets resolution; PRF sets unambiguous range. They are **two separate trades**, which is why real radars change both by mode — search, track, engagement

Beyond the Unambiguous Range: The Ghost

A radar with $PRF = 10$ kHz has $PRI = 100\ \mu$s, so $R_{\text{unamb}} = 15$ km. A target sits at **18 km**.

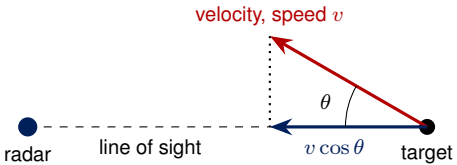


- The echo needs $2(18\text{ km})/c = 120\ \mu$s, so it arrives $20\ \mu$s *after* pulse 2 went out
- The radar times every echo from its **most recent** pulse: $c\,(20\ \mu\text{s})/2 = 3$ km — a **ghost**
- **Reported range** = **true range** $- R_{\textbf{unamb}} = 18 - 15 = 3$ km (subtract again if still beyond it)

Doppler Shift: Measuring Velocity

$$f_{\text{return}} = f_{\text{original}} \left(1 \pm \frac{2v\cos\theta}{c}\right)$$

- A moving target shifts the echo's frequency: $+$ for a **closing** target, $-$ for an **opening** one
- v is the target's **speed** in m/s — never negative. The 2 is the two-way path again
- θ is the **acute angle** between the target's velocity and the radar's line of sight
- *Across* the beam ($\theta = 90°$): **no** shift at all



Closing shown ($+$). Opening: v points away from the radar ($-$); θ is still the acute angle.

The **Doppler shift** is the change, in Hz: $\Delta f = 2v\cos\theta\, f_{\text{original}}/c$, so $f_{\text{return}} = f_{\text{original}} \pm \Delta f$

Method: Pulse-Doppler Calculations

1. **Convert the givens**: times to s, frequencies to Hz, speeds to m/s (km/h $\div$ 3.6), miles to km ($\times$ 1.61)
2. **Timing to range**: $PRI = 1/PRF$, then $\Delta R = c\tau/2$ and $R_{\text{unamb}} = c\,PRI/2$
3. **Beyond R_{unamb}?** The radar reports the true range minus R_{unamb} — a ghost
4. **Doppler shift**: $\Delta f = 2v\cos\theta\, f_{\text{original}}/c$, with θ the acute angle off the line of sight
5. **Direction sets the sign**: $f_{\text{return}} = f_{\text{original}} + \Delta f$ closing, $f_{\text{original}} - \Delta f$ opening. Backward: a return *above* f_{original} means closing, and $v = c\,|\Delta f|/(2f_{\text{original}}\cos\theta)$
6. **Check**: ΔR is far smaller than R_{unamb}; the shift is tiny next to the carrier, so f_{return} is reported to the **nearest hertz**

Find Δf first. Then add it (closing) or subtract it (opening).

Worked Example

A ground radar's antenna is **15 ft** above ground level (AGL). It must cover targets flying up to **500 ft** AGL. It transmits at $f_{\text{original}} = 1$ GHz with pulse width $\tau = 1$ μs.

1. What PRF makes the unambiguous range equal to the line-of-sight range?
2. What is the range resolution? Two inbound aircraft fly 20 m apart, nose to tail — does the radar see one return or two?
3. A target **closes** at 1440 km/h, its velocity $\theta = 30^\circ$ off the line of sight. Find the Doppler shift and the return frequency
4. **Working backward** (a different radar): a 300 MHz radar receives 299.999875 MHz from a target with $\theta = 60^\circ$. Find its speed and direction

Worked Example — Solution: The Waveform

1. PRF from the horizon (Method steps 1–2)
Line of sight, from the sheet's row (Lesson 32) — heights in feet, answer in miles:

$$R_{LOS,mi} = \sqrt{2(15)} + \sqrt{2(500)} = 37.1 \text{ mi}$$

$$37.1 \text{ mi} \times 1.61 \ \tfrac{\text{km}}{\text{mi}} = 59.73 \text{ km}$$

Set $R_{\text{unamb}} = 59.73$ km and solve for the PRI:

$$PRI = \frac{2R_{\text{unamb}}}{c} = \frac{2(59{,}730 \text{ m})}{3 \times 10^8 \text{ m/s}} = 398.2 \ \mu\text{s}$$

$$PRF = \frac{1}{398.2 \ \mu\text{s}} = 2.51 \text{ kHz}$$

This is the **highest** PRF that stays unambiguous out to the horizon. A lower one works too — but nothing beyond the horizon can be seen, so more unambiguous range buys nothing.

2. Range resolution

$$\Delta R = \frac{c\tau}{2} = \frac{(3 \times 10^8)(1 \times 10^{-6})}{2} = 150 \text{ m}$$

$20 \text{ m} < 150 \text{ m}$: the two echoes merge, so the radar sees **one** return.

Check: 150 m is far smaller than 59.73 km, as it must be when τ is far shorter than the PRI.

Worked Example — Solution: Doppler

3. Closing target (Method steps 1, 4, 5)
1 km/h is 1000 m in 3600 s, so divide by 3.6:

$$v = \frac{1440 \text{ km/h}}{3.6} = 400 \text{ m/s}$$

$$\Delta f = \frac{2(400)\cos 30^\circ}{3 \times 10^8}\,(1 \times 10^9) = 2309 \text{ Hz}$$

Closing, so **add** it to $1{,}000{,}000{,}000$ Hz:

$$f_{\text{return}} = 1{,}000{,}002{,}309 \text{ Hz} = 1.000002309 \text{ GHz}$$

4. Working backward

$$\Delta f = 299{,}999{,}875 - 300{,}000{,}000 = -125 \text{ Hz}$$

The return is *below* f_{original}, so the target is **opening**.

$$v = \frac{c\,|\Delta f|}{2 f_{\text{original}} \cos\theta} = \frac{(3 \times 10^8)(125)}{2(300 \times 10^6)\cos 60^\circ}$$

$$v = 125 \text{ m/s}$$

Answer: **125 m/s, opening**.

2309 Hz on 1 GHz is about two parts per million. Round $2v\cos\theta/c$ early and the answer is gone — carry every digit and report to the nearest hertz.

Your Turn

A surveillance radar operates at $f_{\text{original}} = 3$ GHz with $\tau = 2$ μs and $PRF = 800$ Hz.

1. What is the range resolution?
2. What is the maximum unambiguous range?
3. A target **opens** at 900 km/h, its velocity 60° off the line of sight. Find the Doppler shift and the return frequency to the nearest hertz
4. Another return arrives 3600 Hz *above* f_{original} from a target flying straight along the line of sight ($\theta = 0^\circ$). Find its speed and direction
5. A third target is at a true range of 250 km. Where does the radar report it?

Wrap-Up

Key ideas:

- $PRI = 1/PRF$; range from round-trip time, $R = c\,\Delta t/2$
- **Range resolution** $\Delta R = c\tau/2$ — set by **pulse width**
- **Unambiguous range** $R_{\text{unamb}} = c\,PRI/2$ — set by **PRF**. A target beyond it is reported R_{unamb} too close: a ghost
- **Doppler**: $f_{\text{return}} = f_{\text{original}}(1 \pm 2v\cos\theta/c)$ — $+$ closing, $-$ opening, none when crossing

Next lesson:

- **Lesson 36 — Objective 3.8: Signal-to-Noise Ratio.** Why a weaker echo costs detection range, and what “minimum detectable signal” really means
- **Quiz 9** (Objectives 3.3–3.5) is due tonight