ECE 447 Fall 2025

Lesson 12

Angle Modulation, Part 2

UNITED STATES AIR FORCE ACADEMY

Schedule and Admin

- Schedule
- Admin
  - **Lab 3 Assignment**. Due Lesson 14 - specifically 15 Sep by 2359 via Gradescope upload.
  - HW2 posted on course website
  - Skills Review graded. HW1 Lathi solutions posted to Teams - getting your submissions graded...
  - GR1 is a week from this Friday. Review day the Wednesday prior. Topics will primarily be from Chapter 4 of the textbook, though topics from Chapters 2-3 are fair game.

Review

- What are k_f and k_p?

- FM/PM: Problem 4.5-2 in textbook

Angle Modulation Bandwidth

- Can't directly apply Fourier transform properties to angle modulated signals for bandwidth analysis. Why?
- Instantaneous Frequency vs. Spectral Frequency
  - They are NOT the same thing
  - Instantaneous frequency is a *time-varying* representation of the frequency
  - Spectral frequency is the power of a signal vs. frequency
  - This is what the textbook refers to as "the fallacy in the reasoning of the pioneers"
- We will first consider NBFM and WBFM, then PM
- We won't work through entire derivations - see textbook for those

Narrowband FM (and PM)

- Recall for FM: $\varphi_{FM}(t) = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha]$
- Let $a(t) = \int_{-\infty}^{t} m(\alpha)d\alpha$
- Expanding the FM signal in a power series & only keeping the first two terms gives us our NBFM signal (see 4.6 in textbook)
- For NBFM, $|k_f a(t)| \ll 1$ so that all power series terms except the first two are negligible
- Approximated FM:

$$\varphi_{FM}(t) \approx A[\cos(\omega_c t) - k_f a(t)\sin(\omega_c t)]$$

- Approximated PM:

$$\varphi_{PM}(t) \approx A[\cos(\omega_c t) - k_p m(t)\sin(\omega_c t)]$$

Narrowband FM (and PM)

$$\varphi_{FM}(t) \approx A[\cos(\omega_c t) - k_f a(t) \sin(\omega_c t)]$$

- Let $m(t)$ and, thus, $a(t)$ be band-limited to B
- Then $\varphi_{FM}(t)$ has an approximate bandwidth of $2B$ Hz - why?

- Similarly, $\varphi_{PM}(t)$ also has an approximate bandwidth of $2B$ Hz

- Comparison to AM
  - Both are linear with BW of $2B$
  - AM: modulation is "in-phase" – amplitude varies
  - NBFM: modulation is "in-quadrature" – angle varies
  - NBFM has no advantage over AM, but can be used to generate WBFM which does have a noise advantage over AM

WIDEBAND FM

- Derivation in 4.6.1 in textbook - we'll jump to the highlights
- WBFM *spectrum* bandwidth: $B_{FM} = 2\left(\frac{k_f m_p}{2\pi} + B\right)$, where m_p is the peak message amplitude and B is the message bandwidth
- $\pm k_f m_p$ is the carrier frequency deviation in rad/s
- *Peak frequency deviation* in Hz: $\Delta f = k_f \frac{m_{max} - mmin}{2 \cdot 2\pi}$
- Deviation ratio (similar to modulation index for tone modulation): $\beta = \frac{\Delta f}{B}$
- **Carson's Rule**:

$$B_{FM} = 2(\Delta f + B) = 2B(\beta + 1)$$

PM

- Many of the same results from FM apply to PM
- *Peak frequency deviation* in Hz: $\Delta f = k_p \frac{\dot{m}_p}{2\pi}$
- $B_{PM} = 2(\Delta f + B) = 2B(\beta + 1)$
- Comparison with FM
  - FM: Δf only depends on m_p (independent of message spectrum)
  - PM: Δf depends on the peak value of $\dot{m}(t)$, which is strongly dependent on the message spectrum
  - PM: When $m(t)$ is concentrated at lower frequencies, B_{PM} will be smaller than when $m(t)$ is concentrated at higher frequencies
  - See Examples 4.8 and 4.9 in textbook (and HW Problem 4.6-6)