ECE 447 Fall 2025

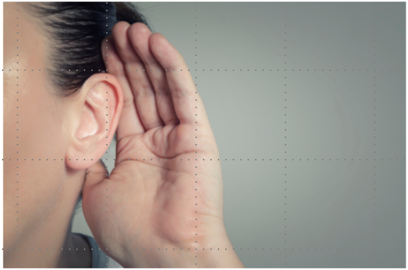
Lesson 05 Analysis and Transmission of Signals

UNITED STATES AIR FORCE ACADEMY

Life/Leadership Lesson of the Day

- Listen!
- You aren't going to learn anything if you are the one doing all of the talking
- Plus, you run the risk of showing off what you don't know

- Good relationship advice, too!



Schedule and Admin

- Schedule
- Admin
  - **Lab 1 Assignment**. The assignment associated with Lab 1 is due Lesson 6 - specifically 21 Aug by 2359 via Gradescope upload. Make sure you submit a **narrowband FM** signal - not a wideband stereo FM radio signal.
  - **Future HW problems**...

Fourier Transform Pair

Synthesis (*Inverse Fourier Transform*):

$$x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{+j2\pi ft} df$$

Analysis (just *Fourier Transform*):

$$X(f) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

When using angular frequency:

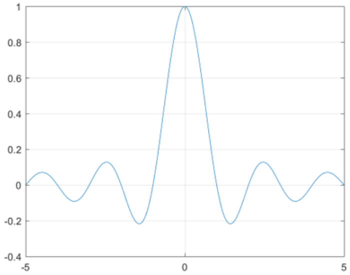
$$X(\omega) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) e^{+j\omega t} d\omega$$

Fourier Transform Example

Rectangle Function, $\Pi(t/\tau)$

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \\ &= \int_{-\tau/2}^{+\tau/2} 1 e^{-j2\pi f t} dt \\ &= \frac{-1}{j2\pi f} \left(e^{-j2\pi f t} - e^{j2\pi f t} \right) \Big|_{-\tau/2}^{+\tau/2} \\ &= \frac{\sin(\pi f \tau)}{(\pi f)} \\ &= \tau \mathrm{sinc}(f\tau) \end{aligned}$$



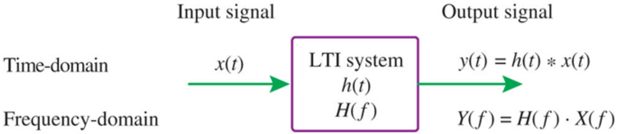
Bandwidth?

Fourier Transform Pairs and Properties

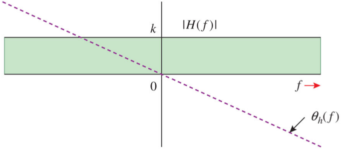
Tables 3.1 and 3.2 in textbook

- $\delta(t) \Leftrightarrow$
- $\delta(f - f_0) \Leftrightarrow$
- $\cos(2\pi f_0 t) \Leftrightarrow$
- $\Lambda(t) \Leftrightarrow$
- $C \Leftrightarrow$
- $g(at) \Leftrightarrow$
- $g(t)e^{j2\pi f_0 t} \Leftrightarrow$
- $g_1(t) * g_2(t) \Leftrightarrow$

LTI Systems

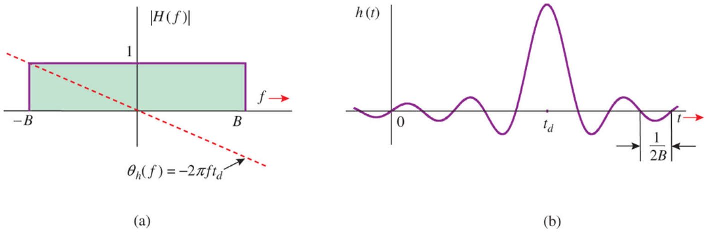


- $H(f) =$
- $|Y(f)| =$, $\theta_y(f) =$
- Distortionless transmission: $y(t) = k \cdot x(t - t_d)$
- $|H(f)| =$, $\theta_h(f) =$

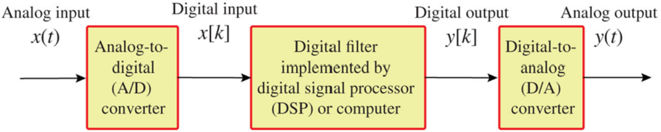


Filters

- Why can't we build ideal filters?



- SDRs → Digital filters → DSP and Lab 2!



Channel Effects

- Linear distortion (amplitude or phase)
    - Spreads digital pulses out
    - Creates Inter Symbol Interference (ISI), or when a digital pulse smears outside of its time slot
- "Fading"
    - Large scale fading: changes in P_R based on stable conditions; caused by shadowing, blocking from buildings, topography
    - Small scale fading: rapid changes in P_R relative to signal freq; caused by multipath, mobility

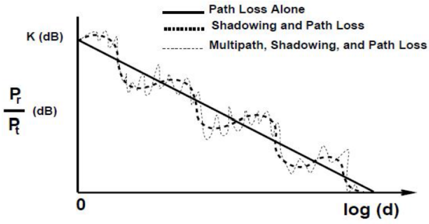


Fig.1.1 Path loss, shadowing and multipath versus distance

Parseval's Theorem and ESD

- Parseval's Theorem:

$$E_g = \int_{-\infty}^{\infty} |G(f)|^2 \, df$$

- Energy Spectral Density (ESD):

$$\Psi_g(f) = |G(f)|^2 \quad \rightarrow \quad E_g = \int_{-\infty}^{\infty} \Psi_g(f) df$$

- Use to determine essential bandwidth B (e.g., B that contains 95% of E_g - see Ex. 3.21... and HW problem 3.7-5)
- Modulation shifts ESD up and down, just like with a signal's spectrum; total energy is halved

PSD and Autocorrelation

- Power Spectral Density (PSD):

$$S_g(f) = \lim_{T\to\infty} \frac{|G(f)|^2}{T},$$

where $G_T(f)$ is a truncated part of the power signal $G(f)$

- Time average of ESD of $g(t)$
- Can also take Fourier Transform of the signal's *time autocorrelation* function $\mathcal{R}_g(\tau)$:

$$S_g(f) = \mathcal{F}[\mathcal{R}_g(\tau)]$$

- What is time autocorrelation? Next slide...

Time Autocorrelation

- Defined as:

$$\mathcal{R}_g(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)g^*(t-\tau)dt$$

- This is an **average over time**, not a statistical average (that comes later in the course)
- Similar to convolution without the flip