ECE 447 Fall 2025

Lesson 35

Binary System Performance, Part 1

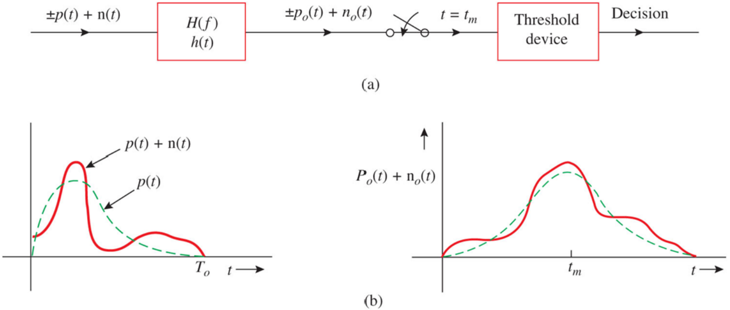
Schedule and Admin

- Schedule.
  - Lesson 35 - Matched filters, binary digital system performance (Part 1)
  - Lesson 36 - Binary digital system performance (Part 2)
  - Lesson 37 - Error correction
  - Lesson 38 - MATLAB Lab 7: Matched filters, multi-path, OFDM, BER (substitute for Lt Col Booth TBD)
  - Lesson 39 - Advanced topics: OFDM, MIMO, CDMA
  - Lesson 40 - Course review
- Admin
  - **HW8**. Assigned today. Due 02 Dec (Lsn 39) to Gradescope.

Binary Communication Systems

- Received signal is $y(t) = \pm p(t) + n(t), 0 \leqslant t \leqslant T_b$
- Receiver filters signal through $H(f)$: $r(t) = h(t) * y(t) = \pm p_0(t) + n_0(t)$

  (ideally the filter aligns $r(t)$ so it is sampled at the "widest opening of the eye")
- Decision variable: $r(t_m) = \pm p_0(t_m) + n_0(t_m)$, where $n_0(t_m) \sim \mathcal{N}(0, \sigma_n^2)$
- Let $\pm p_0(t_m) = \pm A_p$. Then $r_0(t) \sim \mathcal{N}(-A_p, \sigma_n^2)$ and $r_1(t) \sim \mathcal{N}(A_p, \sigma_n^2)$
- Optimum detection threshold? Probability of bit error P_e?

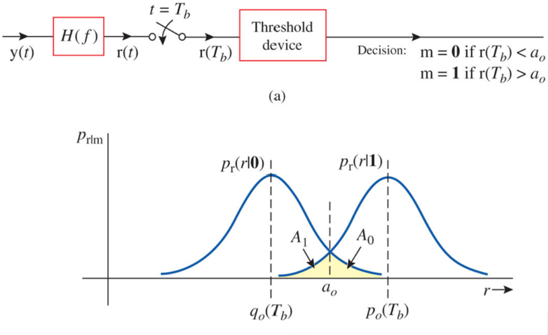


Matched Filter

- Want to minimize P_e using best possible receiver filter $H(f)$
- Did you follow the matched filter derivation in the reading (Section 9.1.2)?
- Cauchy-Schwarz inequality!
- General matched filter: $H(f) = \frac{P^*(f)e^{-j2\pi f t_m}}{S_n(f)}$
- For AWGN:
    - $S_n(f) = N_0/2$ where N_0 (or $\mathcal{N}$ in the textbook) is the noise PSD (or a measure of the total noise power in 1 Hz of bandwidth)
    - $H(f) = P^*(f)e^{-j2\pi f t_m}$ and $h(t) = p(t_m - t)$ for any real valued pulse $p(t)$
    - t_m becomes T_0 since $p_0(t) = h(t) * p(t)$ reaches its max amplitude at $t = T_0$
    - $H(f) = P^*(f)e^{-j2\pi f T_)}$ and $h(t) = p(T_0 - t)$
    - $P_e = Q\left(\sqrt{\frac{2E_p}{N_0}}\right)$, where $E_p = \int_0^{T_0} |p(t)|^2 dt$ is the energy of the pulse
- Correlation detector produces equivalent result as matched filter - uses cross-correlation to measure similarity of received signal with $p(t)$

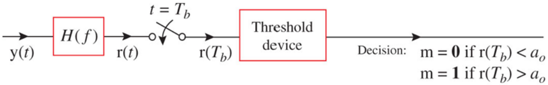
Binary Signaling

- Generalized form of what we've already covered - allow two different transmit pulses, $p(t)$ or $q(t)$ (representing **1** or **0**)
- Both have same noise $n(t)$ added to them
- $p_{r|m} \sim$ Gaussian with mean either $q_0(T_b)$ or $p_0(T_b)$ and variance σ_n^2
- Optimal threshold: $a_0 = \frac{p_0(T_b)+q_0(T_b)}{2}$ (assuming equally likely transmission, i.e., $p_m(0) = p_m(1) = 0.5$)



Binary Signaling (cont'd)

- Optimum filter: $H(f) = [P^*(f) - Q^*(f)]e^{-j2\pi f T_b}$ and $h(t) = p^*(T_b - t) - q^*(T_b - t) \quad \rightarrow$ matched to pulse $p(t) - q(t)$
- $P_e = P_b = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N_0}}\right)$, where $E_{pq} = \text{Re}\left\{\int_0^{T_b} p(t)q^*(t)dt\right\}$
- For real values pulses, $h(t) = p(T_b - t) - q(T_b - t)$



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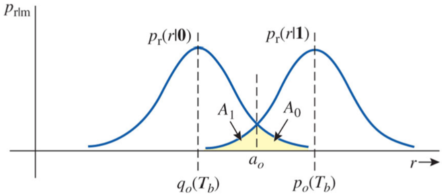
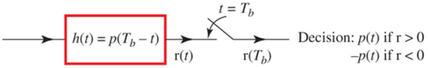


Binary Signaling Performance (AWGN)

Polar signaling, $q(t) = -p(t)$



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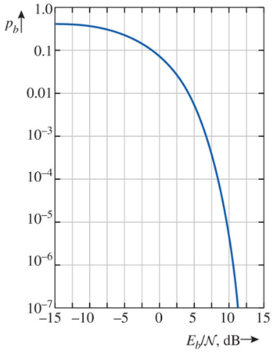

(b)