ECE 447 Fall 2025

Lesson 31

Random Processes

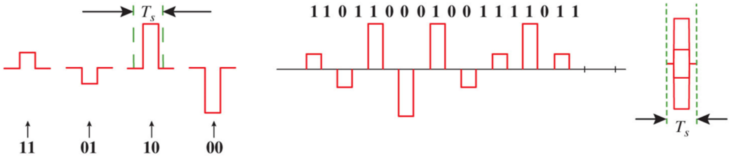
UNITED STATES
AIR FORCE
ACADEMY

Schedule and Admin

- Course website down. :(
- Schedule update.
  - Lesson 32 - SDR Lab 6: ADS-B digital signals.
  - Lesson 33 - **Review Day**
  - Lesson 34 - GR2 (Thursday, 13 Nov)
  - Lessons 35-37 - Binary Digital System Performance, Error Correction
  - Lesson 38 - MATLAB Lab 7: Matched filters, multi-path, OFDM, BER (substitute for Lt Col TBD)
  - Lesson 39 - Advanced topics: OFDM, MIMO, CDMA
  - Lesson 40 - Course review
- Admin
  - **Lab 5**. PDF due 6 Nov (Lsn 32) to Gradescope.
  - **HW6**. Due 4 Nov (Lsn 31) to Gradescope.
  - **HW7**. Due 10 Nov (Lsn 33) to Gradescope.

Review: M-ary Baseband Signaling

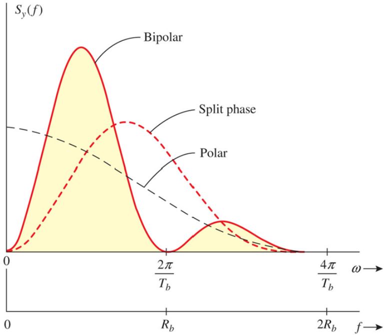
- Assign multiple bits to each pulse - to transmit n bits per pulse, you need $M = 2^n$ pulses (which is where the M in M-ary comes from)
- M-ary PAM: amplitudes define the pulses (other types of M-ary)
- Not a free lunch! See Example 6.6 in textbook - 4-ary PAM results in the same bandwidth as binary, but requires **5x** the original signal power
- New term! **Baud rate**: pulse or symbol rate. Ex: if using 16-ary PAM at a baud rate of 250 MBd (mega baud), the bit rate is 1 Gbps
- Baud rate focuses on physical transmission speed and should be used with Nyquist's theorem (2 bps/Hz → 2 baud/s/Hz)



(a) (b) (c)

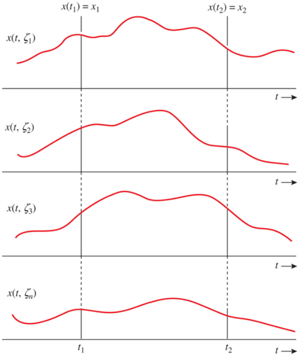
Review

- Baseband signal bandwidth is inversely related to what?
- Rate of information transfer involves what?
- The figure on the right is for what type of line code?
- How does the bandwidth change for "full-width" pulses?



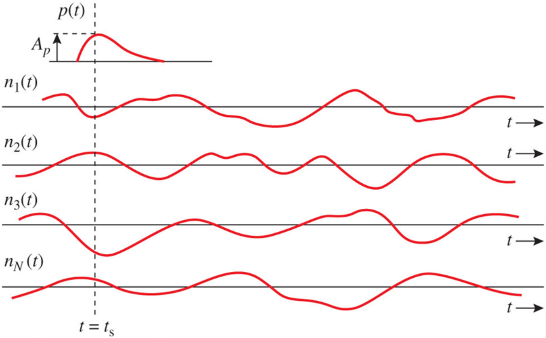
Random Variables to Random Processes

- Random process: a RV that is a function of time (aka stochastic process)
- RV: number of cars on a specified length of I-25 at a specific time every day
- RP: number of cars on a specified length of I-25 at every time throughout a day
- Instead of a collection of sample points mapped to real numbers (RV), we have a collection of sample *functions* that make up the entire sample space, i.e. a RP **ensemble**



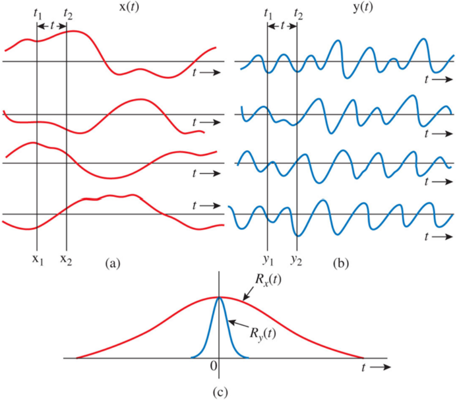
Describing RPs

- Can't always describe RP analytically - have to derive statistics from experimental data
- Complete info of multiple dependent RVs given by joint PDF
- Usually just want first- and second-order statistics
- **Ensemble statistics:** hold at a constant value of time and take statistics across RP ensemble at that time
- Example: threshold detection of polar signal



Spectral Content of RPs

- Autocorrelation again!
- $R_x(t_1, t_2) = \overline{x_1 x_2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 p_x(x_1, x_2; t_x, t_2) dx_1 dx_2$
- Will you have to use this? Not explicitly in this class
- How do we get the PSD from the autocorrelation function?

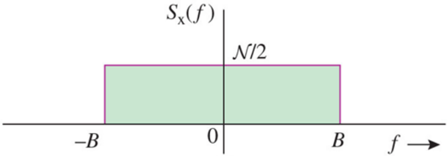


Categories of RPs

- **Stationarity**: a stationary RP's statistical characteristics do not change with time - $p_x(x;t) = p_x(x)$ and $R_x(t_1, t_2) = R_x(t_2 - t_1) = R_x(\tau)$
- Examples: channel noise is stationary; temperature is not
- **Wide-sense Stationarity**: doesn't meet the strict definition of stationary, but still has mean $\overline{x(t)}$ and autocorrelation $R_x(\tau)$ that are independent of time shifts
- Truly stationary RP do not exist; WSS RPs do exist
- **Ergodicity**: RPs whose ensemble averages (at constant times) are equivalent to the time averages (remember this from Chapter 3?) are WSS ergodic processes
- If process is ergodic, then we only need one sample function to find the ensemble averages

PSD

- Average power of a RP: $P_x = \overline{|x|^2} = R_x(0) = \int_{-\infty}^{\infty} S_x(f)df = 2\int_0^{\infty} S_x(f)df$



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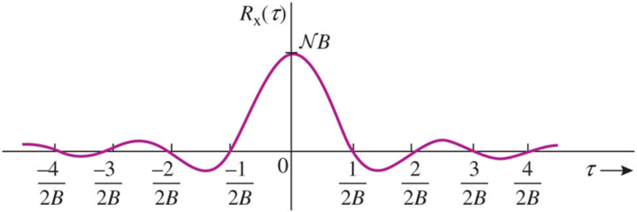

(b)