ECE 447 Fall 2025

Lessons 29

Probability

Schedule and Admin

- Schedule.
- Admin
  - **Lab 4**. Graded. Submit any regrade requests via Gradescope.
  - **HW5**. Graded. Submit any regrade requests via Gradescope.
  - **Lab 5**. PDF due 6 Nov (Lsn 32) to Gradescope.
  - **HW6**. Due 4 Nov (Lsn 31) to Gradescope.

Review

- A binary FSK signal uses carrier frequencies f_A and f_B. The baseband signal uses full-width polar signaling. Estimate the transmitted bandwidth of the FSK signal using Carson's Rule.

Why Probability in Digital Communications?

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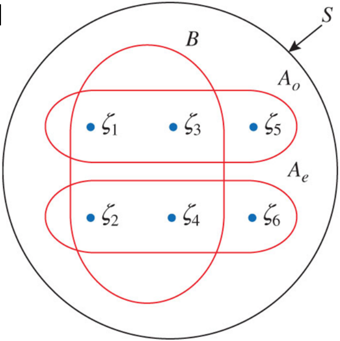
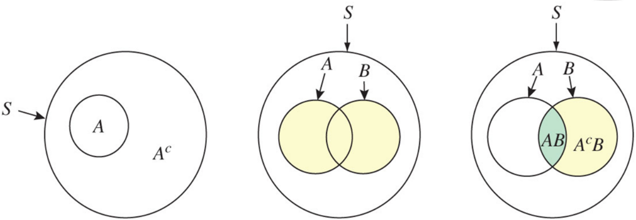
- What did you learn in Math 356 and ECE 346?

Probability Terms & Concept

- Experiment, outcomes, events
- Sample space S, elements, complement
- Union $\bigcup$, intersection $\bigcap$
- Disjoint or mutually exclusive
- Conditional probability, e.g., $P(B|A)$
- Bayes' rule $P(A|B) = \frac{P(A)P(B|A)}{P(A)}$





A^c (a) $A \cup B = B \cup A$ (b) AB and A^cB (c)

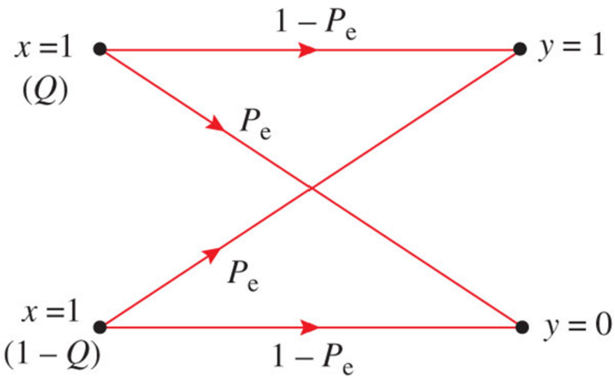
Probability Terms & Concepts (cont'd)

- Independent: iff $P(A \bigcap B) = P(A)P(B)$ (**NOT** the same a mutually exclusive)
- Bernoulli trials
  - $P(k \text{ successes in } n \text{ trials}) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$
  - p is the probability of success of a single trial
  - Example: BSC w/ $P_e = 10^{-3}$. What is the probability that a nibble of a byte is incorrect?
  - Example: Repetition codes; see Example 7.9 in textbook
- Total Probability Theorem: $P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$
- Prior and posterior probabilities, $P(A_i)$ and $P(A_i|B)$
  - Find posterior probability through combination of Bayes' rule and Total Probability Theorem
  - $P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$

Random Variables

- Maps events to real numbers
- Notation
  - $P_{\mathrm{x}}(x_i)$: x is RV, x_i is value RV takes; "Probability of x = x_i"
- Can have multiple RVs or *joint probabilities*
  - $P_{\mathrm{xy}}(x_i, y_j)$: "Probability that x = x_i and y = y_j"
  - If x and y are independent, $P_{\mathrm{xy}}(x_i, y_j) = P_{\mathrm{x}}(x_i)P_{\mathrm{y}}(y_j)$
- Conditional probabilities
  - Probability of x = x_i given y = y_j $\rightarrow P_{\mathrm{x|y}}(x_i|y_j)$
- Marginal probabilities
  - $P_{\mathrm{y}}(y_j) = \sum_i P_{\mathrm{xy}}(x_i, y_j) = \sum_i P_{\mathrm{x|y}}(x_i|y_j)P_{\mathrm{y}}(y_j)$
  - $P_{\mathrm{x}}(x_i) = \sum_j P_{\mathrm{xy}}(x_i, y_j)$

Example: Binary Symmetric Channel (BSC)

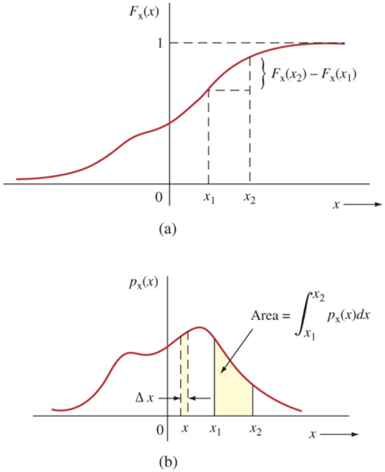


x is the transmitter. The probability of transmitting a 1 is Q. The probability of a bit error due to the channel is P_e. Find the probability of receiving a 1 or 0 at the receiver y.

CDF and PDF

- CDF: $F_{\mathrm{x}}(x) = P(\mathrm{x} \leqslant x)$
  - $F_{\mathrm{x}}(x) \geqslant 0$
  - $F_{\mathrm{x}}(\infty) = 1$
  - $F_{\mathrm{x}}(-\infty) = 0$
  - $F_{\mathrm{x}}(x_1) \leqslant F_{\mathrm{x}}(x_2)$ for $x_1 \leqslant x_2$

- PDF: $p_{\mathrm{x}}(x) = \frac{dF_{\mathrm{x}}(x)}{dx}$
  - $\int_{-\infty}^{\infty} p_{\mathrm{x}}(x)dx = 1$
  - $P(x_1 < x \leqslant x_2) = \int_{x_1}^{x_2} p_{\mathrm{x}}(x)dx$



Gaussian RV

$$p_{\mathrm{x}}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/(2\sigma^2)}$$

- Cover this more in depth next time...