Homework – Navigation and State Estimation

Name: ______________________ **Group Name:** ______________________

Instructions and Grading Policy

Instructions:
This homework assesses your *conceptual understanding* of navigation and state estimation. Questions are intentionally designed to have a perfectly acceptable **short** answer. Longer answers are not automatically better and may indicate unclear understanding.

Submission format:

- Answers must be **typed** (not handwritten).
- For **Q9**, attach screenshots of your MATLAB code and computed output.
- Your submission must clearly include both your **individual name** and **group name** at the top of the document.

You may reference course notes.
You may use GenAI only to help debug (not create) code for Q9. All conceptual responses must be entirely your own.

Grading Policy:

- There are **9 questions total**.
- **Q1–Q8** are worth **1 point each**.
- **Q9 (MATLAB)** is worth **2 points**.
- Responses are graded as either:
  - Meets Standard (full credit), or
  - Does Not Meet Standard (0 points).
- If a question does not meet standard, you may submit **one resubmission per question** to recover full credit.
- **Blank questions, or responses with no clear attempt, are not eligible for resubmission.**
- **Submissions missing an individual name or group name will receive an initial grade of zero, consuming any potential regrade attempts.**
- Total possible points: **10**

Q1. Inertial error growth

Inertial navigation systems compute position by processing accelerometer and gyroscope measurements over time.

Briefly explain why position errors in a pure inertial navigation system tend to grow over time.

Q2. Schuler cycle

Inertial navigation error dynamics are shaped by a fundamental property of near-Earth motion.

What causes the Schuler cycle, and what is its implication for testing inertial drift performance?

Q3. Fusion weighting

Optimal fusion combines information from multiple sensors.

What key factor determines how much weight each sensor receives in an optimal fusion solution?

Q4. Kalman gain

The scalar Kalman filter estimates a single variable.

In one sentence, explain what the Kalman gain represents.

Q5. Operational INS/GPS filter states

Operational INS/GPS navigation filters estimate more states than a basic position and velocity filter.

How many states does a typical operational INS/GPS Kalman filter use, and what are they?

Q6. Nonlinear estimation

Many navigation systems use an Extended Kalman Filter (EKF).

What problem does the EKF solve that the standard linear Kalman filter cannot?

Q7. Hazardous misleading information

Not all navigation failures carry the same operational risk.

Define hazardous misleading information (HMI) and explain why it is a particularly critical concern for navigation in operational military air and space systems.

Q8. Effective sample size

Navigation error datasets are collected as time-series during flight test.

Define correlation time and explain how it produces an effective sample size N_{eff} that is lower than the total number of samples collected.

Q9. MATLAB Exercise – Kalman Filter Update (2 points)

Consider a 4-state navigation vector

$$\mathbf{x} = \begin{bmatrix} x & y & v_x & v_y \end{bmatrix}^\top$$

with prior estimate, measurement, measurement matrix, and measurement covariance

$$\hat{\mathbf{x}}^- = \begin{bmatrix} 100 \\ 200 \\ 10 \\ -5 \end{bmatrix}, \qquad \mathbf{z} = \begin{bmatrix} 104 \\ 197 \end{bmatrix}, \qquad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

Part (a) – Diagonal prior covariance.

$$\mathbf{P}_a^- = \begin{bmatrix} 25 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Using the standard Kalman filter measurement update equations from course notes, compute the innovation $\boldsymbol{\nu}$, Kalman gain $\mathbf{K}$, updated state $\hat{\mathbf{x}}^+$, and updated covariance $\mathbf{P}^+$.

Part (b) – Prior covariance with position–velocity correlation.

$$\mathbf{P}_b^- = \begin{bmatrix} 25 & 0 & 8 & 0 \\ 0 & 25 & 0 & 8 \\ 8 & 0 & 4 & 0 \\ 0 & 8 & 0 & 4 \end{bmatrix}$$

Repeat the update with this $\mathbf{P}_b^-$ and report the same four quantities. This covariance is representative of what a constant-velocity filter produces after several propagation steps with process noise on acceleration.

Part (c) – Compare and explain. In one or two sentences, explain why the velocity states are unchanged in Part (a) but updated in Part (b), even though both parts use the same position-only measurement.

MATLAB hints:

- Define each matrix explicitly before computing.
- Transpose with `'` and use `eye()` for identity matrices.
- Keep your code short and readable.

Your Q9 submission should include:

(a) A screenshot of your complete MATLAB code (both parts)

(b) A screenshot of computed $\boldsymbol{\nu}$, $\mathbf{K}$, $\hat{\mathbf{x}}^+$, and $\mathbf{P}^+$ for Part (a)

(c) A screenshot of computed $\boldsymbol{\nu}$, $\mathbf{K}$, $\hat{\mathbf{x}}^+$, and $\mathbf{P}^+$ for Part (b)

(d) Your typed written response to Part (c)